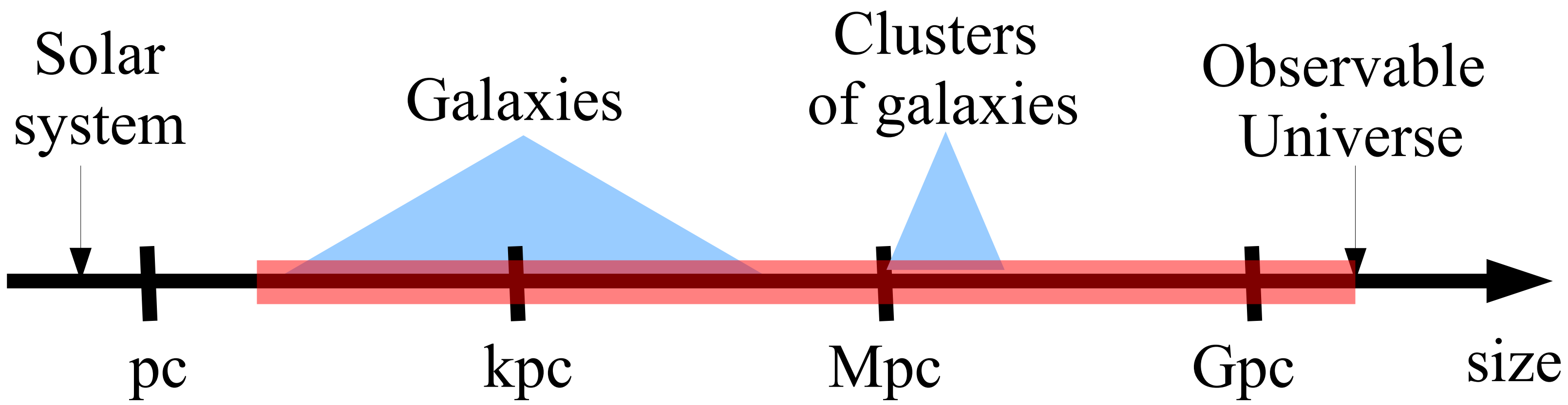
Dark Matter Physics

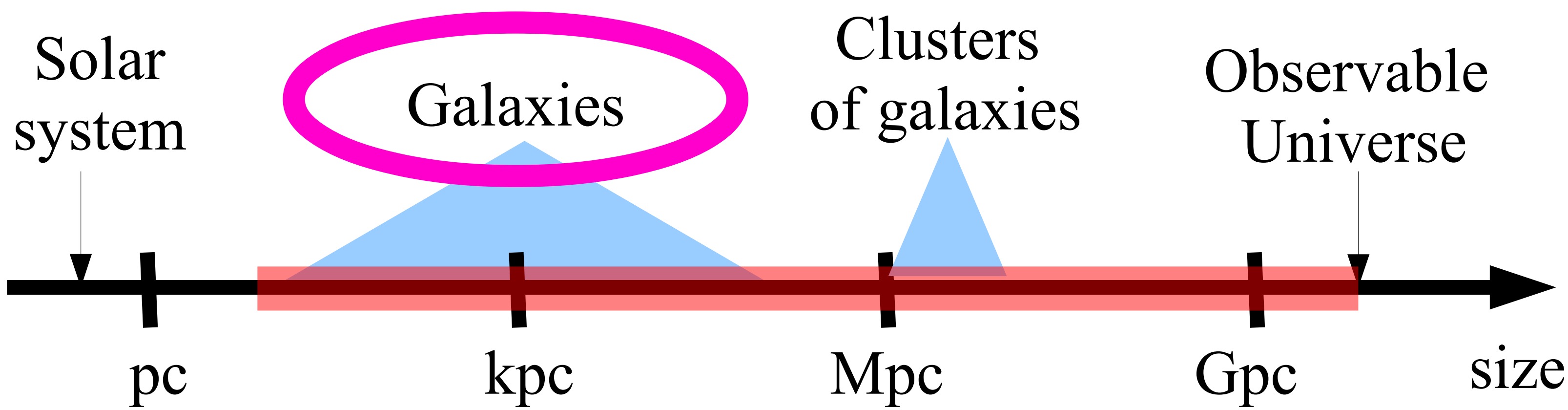
Alejandro Ibarra
Technische Universität München

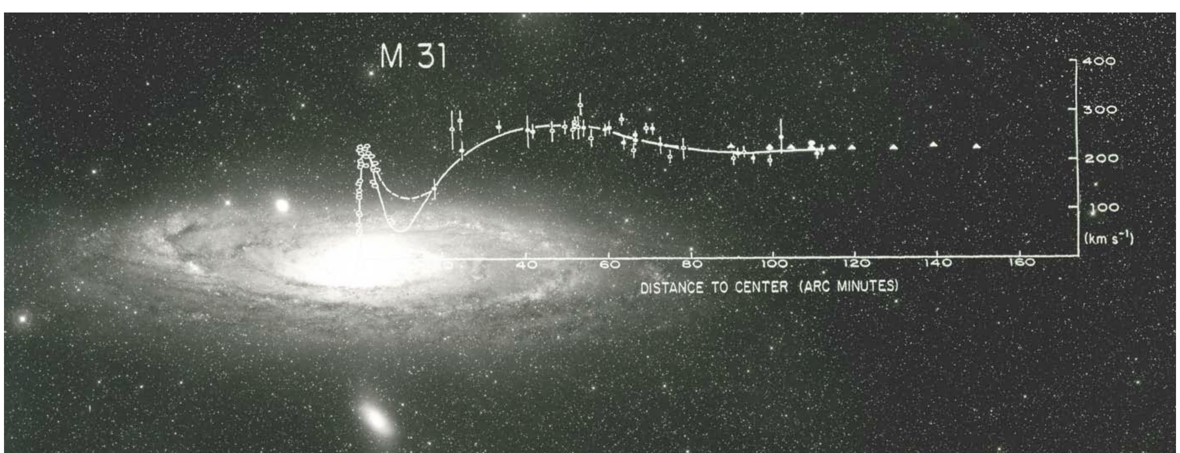
Summer Institute 2016
Xi-Tou
August 2016

There is evidence for dark matter in a wide range of distance scales

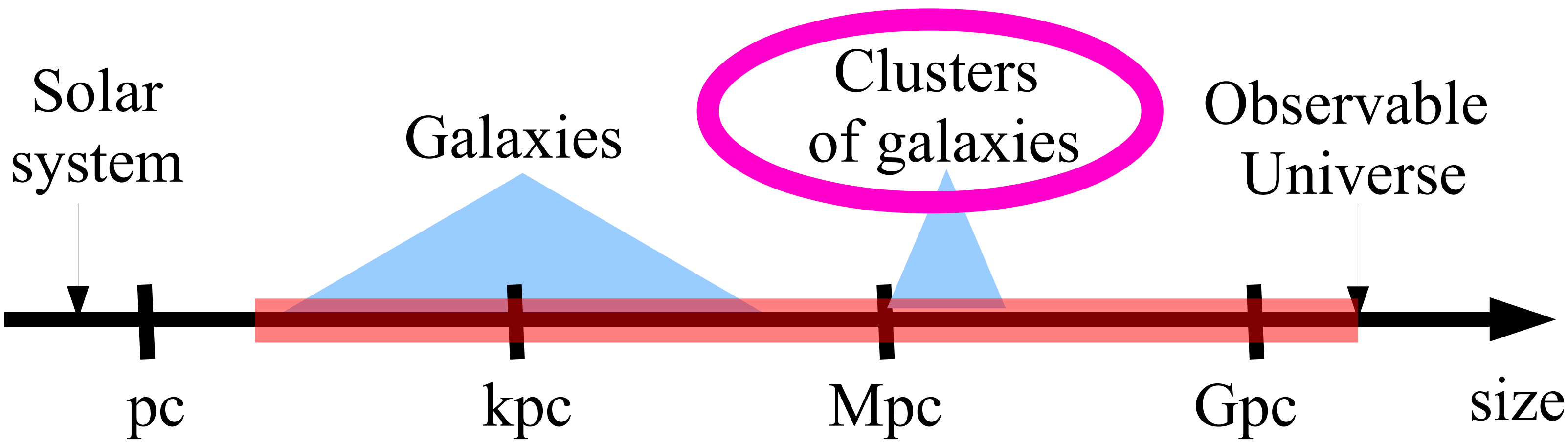


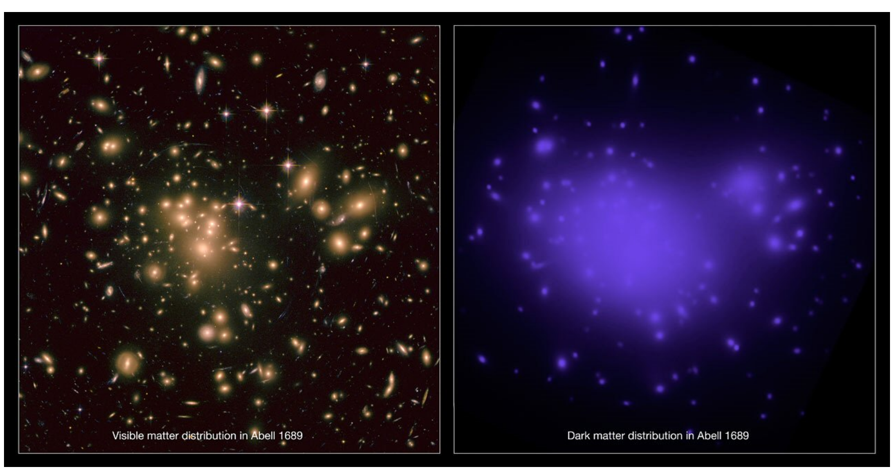
There is evidence for dark matter in a wide range of distance scales



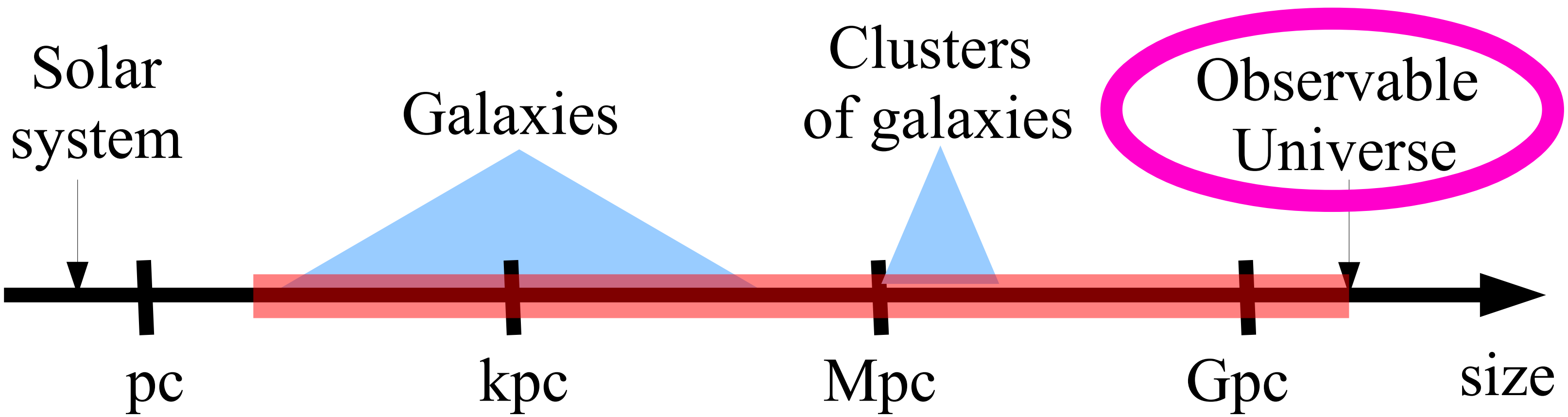


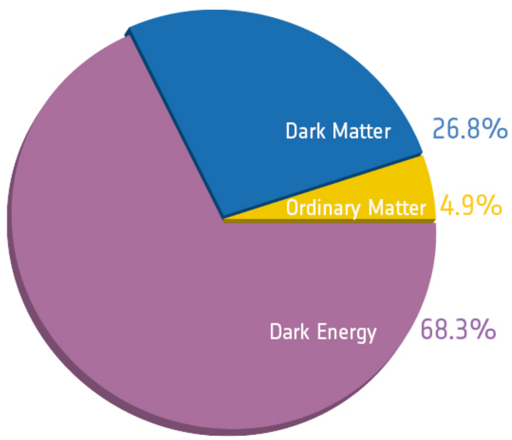
There is evidence for dark matter in a wide range of distance scales

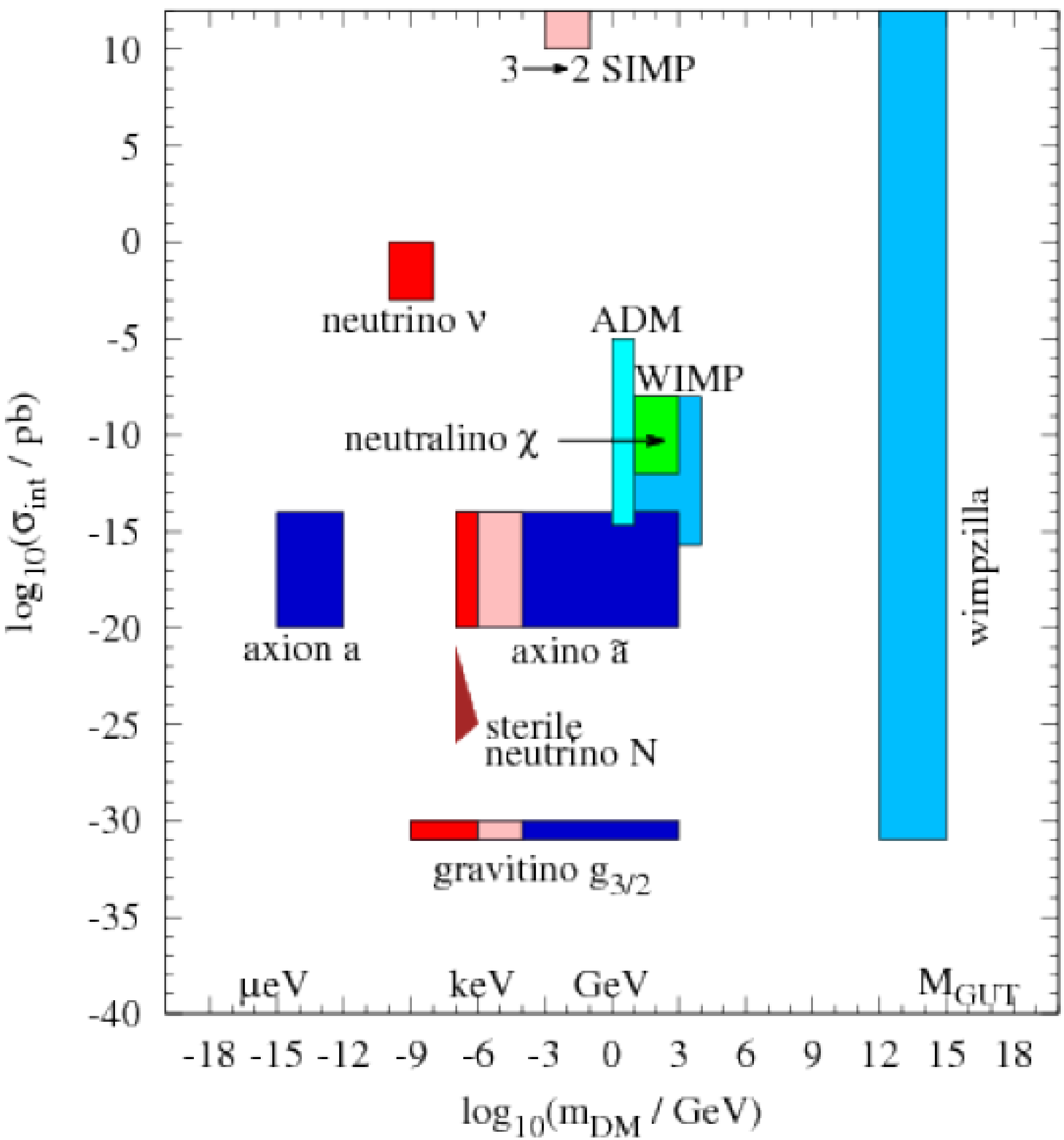




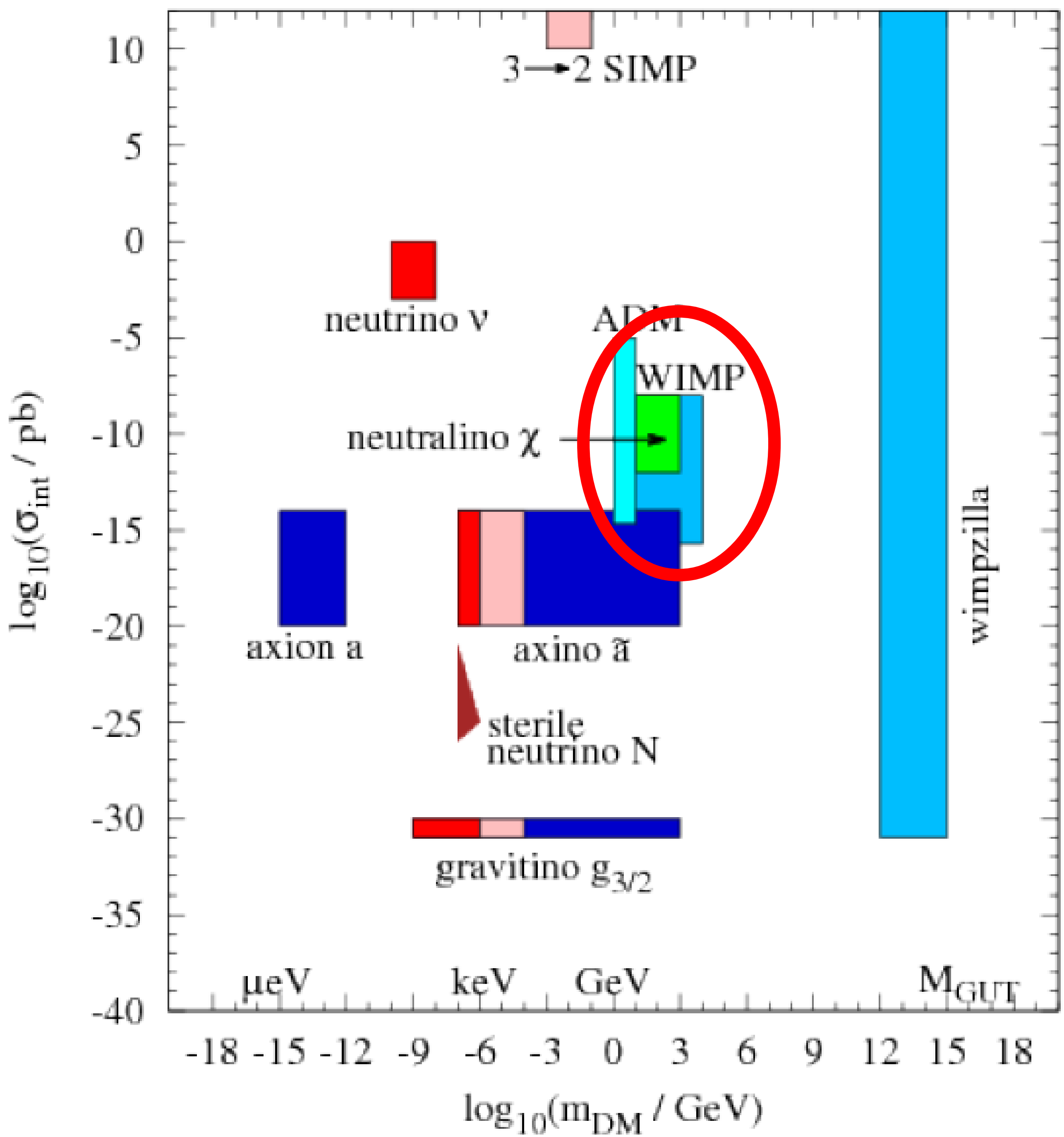
There is evidence for dark matter in a wide range of distance scales







3→2 SIMP
neutrino ν
ADM
WIMP
neutralino χ
wimpzilla
axion a
axino ã
sterile
neutrino N
gravitino $g_{3/2}$
μeV
keV
GeV
M_{GUT}
$\log_{10}(\sigma_{int} / \mathrm{pb})$
$\log_{10}(m_{DM} / \mathrm{GeV})$
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-18
-15
-12
-9
-6
-3
0
3
6
9
12
15
18



3 → 2 SIMP
neutrino ν
ADM
WIMP
neutralino χ
axion a
axino ã
sterile
neutrino N
gravitino $g_{3/2}$
wimpzilla
μeV
keV
GeV
M_{GUT}
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-18 -15 -12 -9 -6 -3 0 3 6 9 12 15 18
$\log_{10}(\sigma_{int}$ / pb)
$\log_{10}(m_{DM}$ / GeV)

Outline

Lecture 1: Evidence for dark matter.

Lecture 2: Dark matter production. Indirect detection.

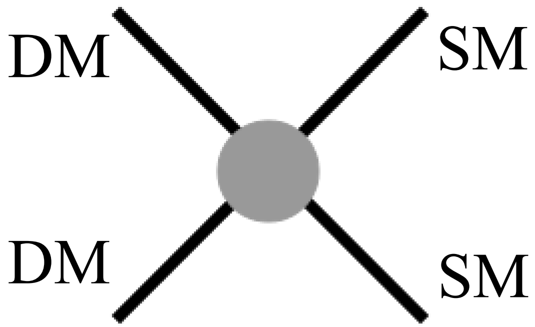
Lecture 3: Indirect detection (cont.), direct detection, collider signals

WIMP history (in a nutshell)

Main assumptions on dark matter WIMPs:

1) The WIMP is stable in cosmological timescales.

2) WIMPs interact in pairs with the Standard Model particles



3) The WIMP interaction strength is *large enough* to keep the DM particles in thermal equilibrium with the SM plasma at very high temperatures.

4) The WIMP interaction strength is *small enough* to allow DM particles to chemically decouple from the SM plasma sufficiently early.

WIMP history (in a nutshell)

WIMP history (in a nutshell)

"yield" = number density of DM particles per comoving volume

$$Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}$$

time

WIMP history (in a nutshell)

"yield" = number density of DM particles per comoving volume

$$Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}$$

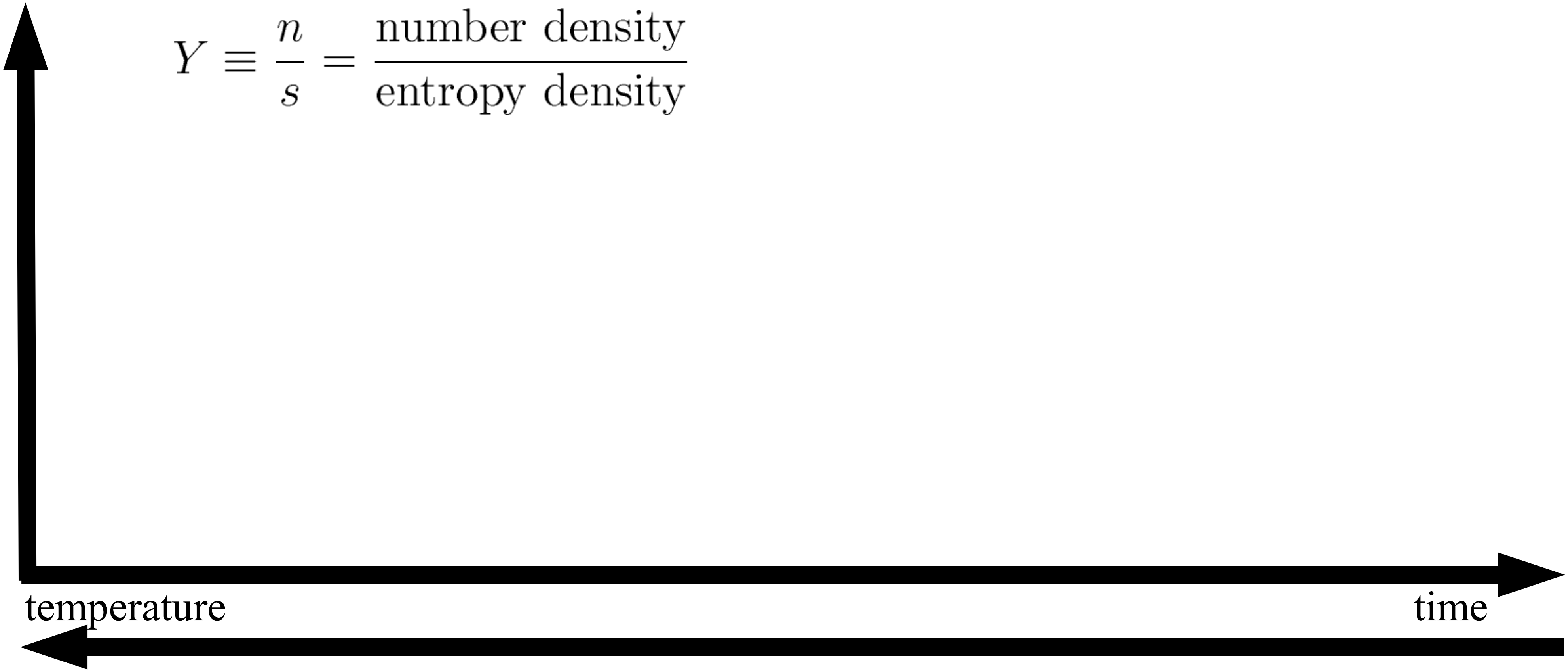
temperature

time

WIMP history (in a nutshell)

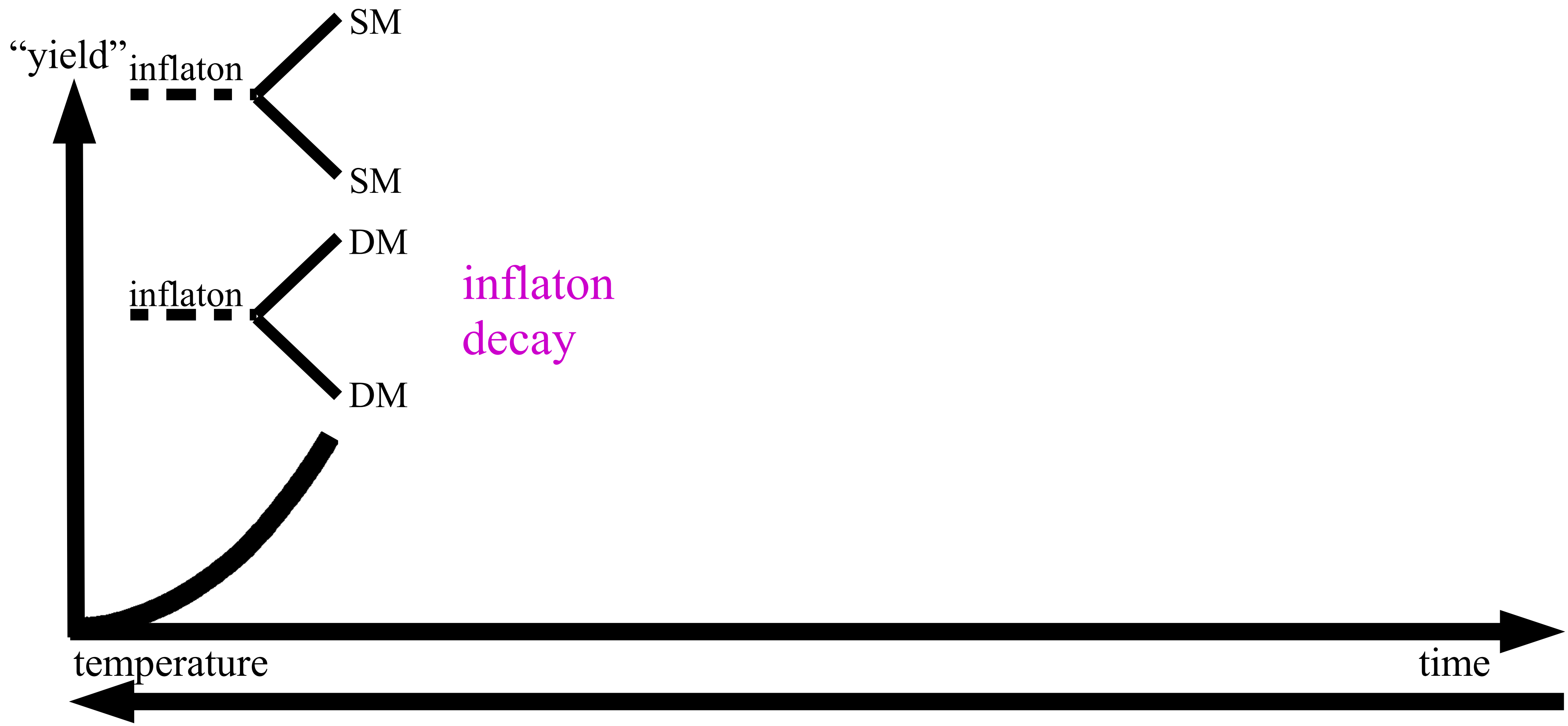
"yield" = number density of DM particles per comoving volume

$$Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}$$



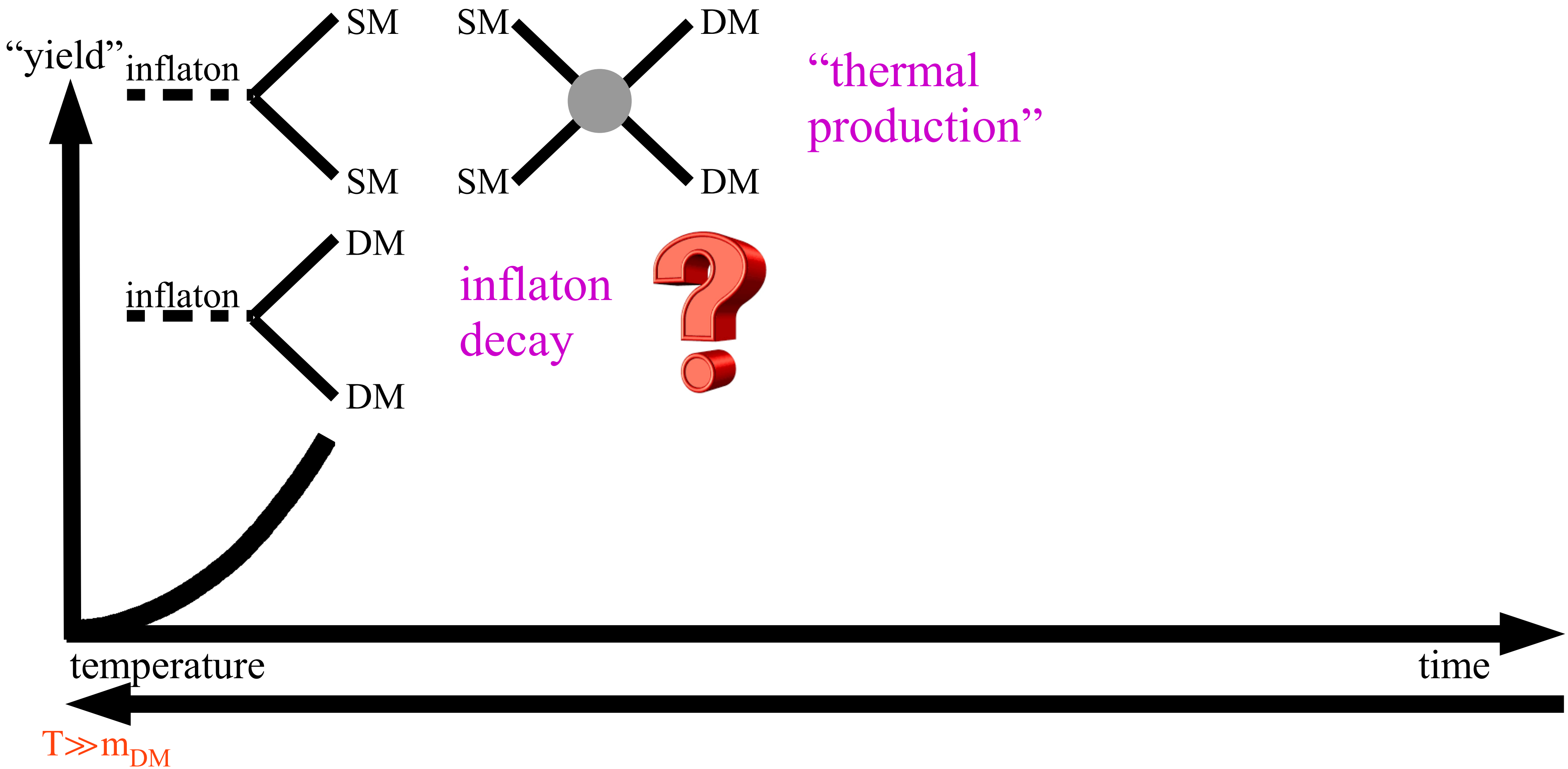
WIMP history (in a nutshell)

WIMP history (in a nutshell)



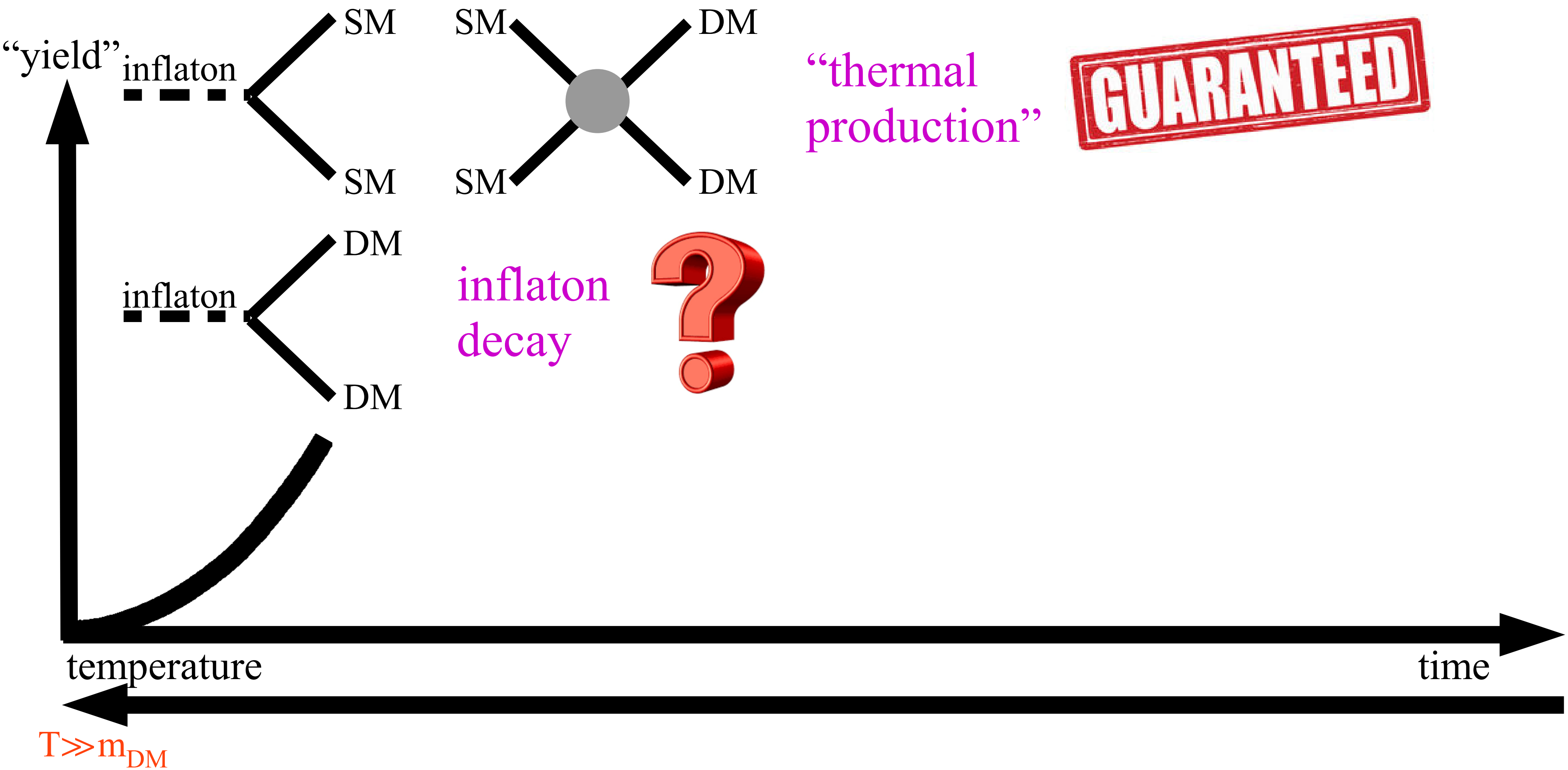
WIMP history (in a nutshell)

WIMP history (in a nutshell)



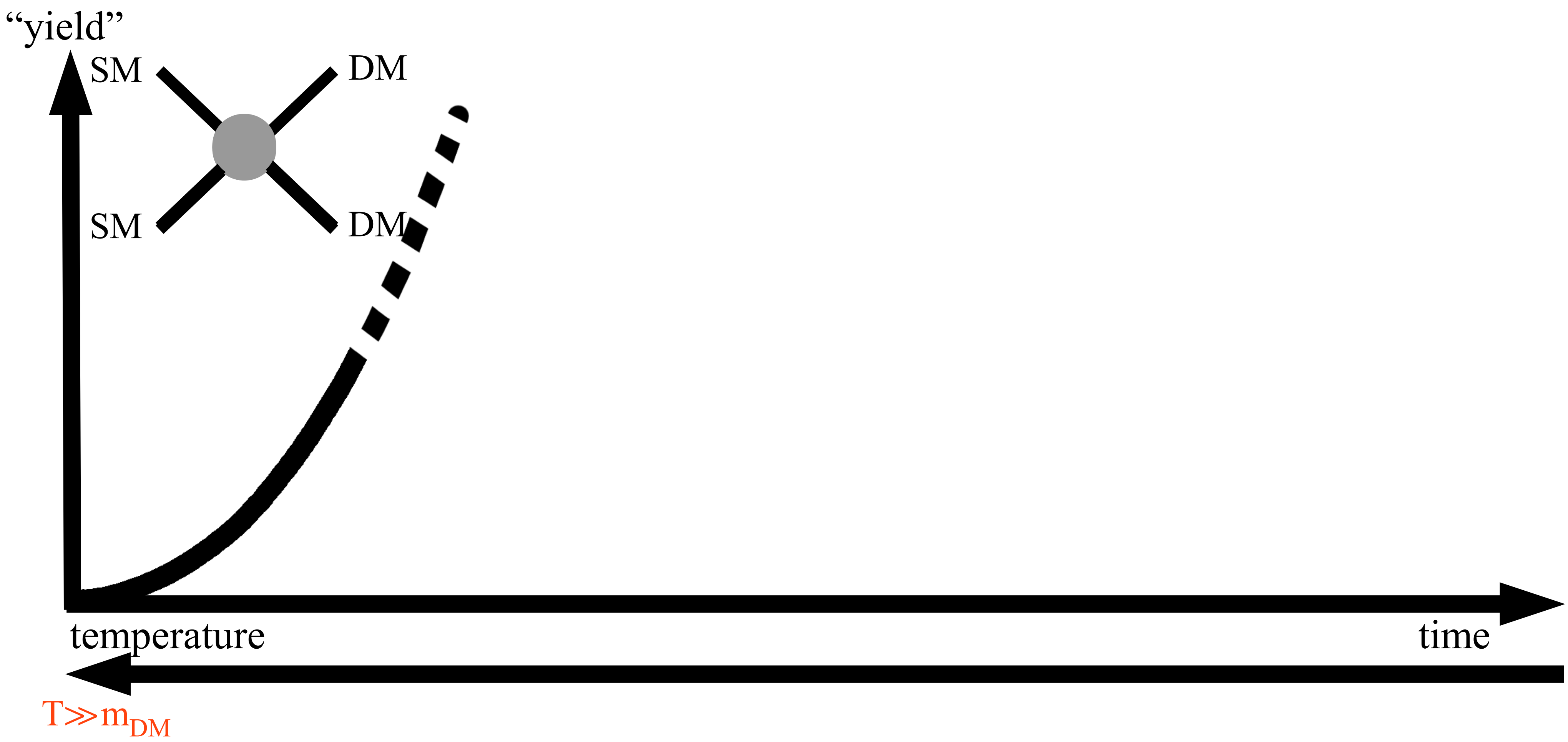
Assume that the temperature of the Universe after reheating was much larger than the DM mass.

WIMP history (in a nutshell)

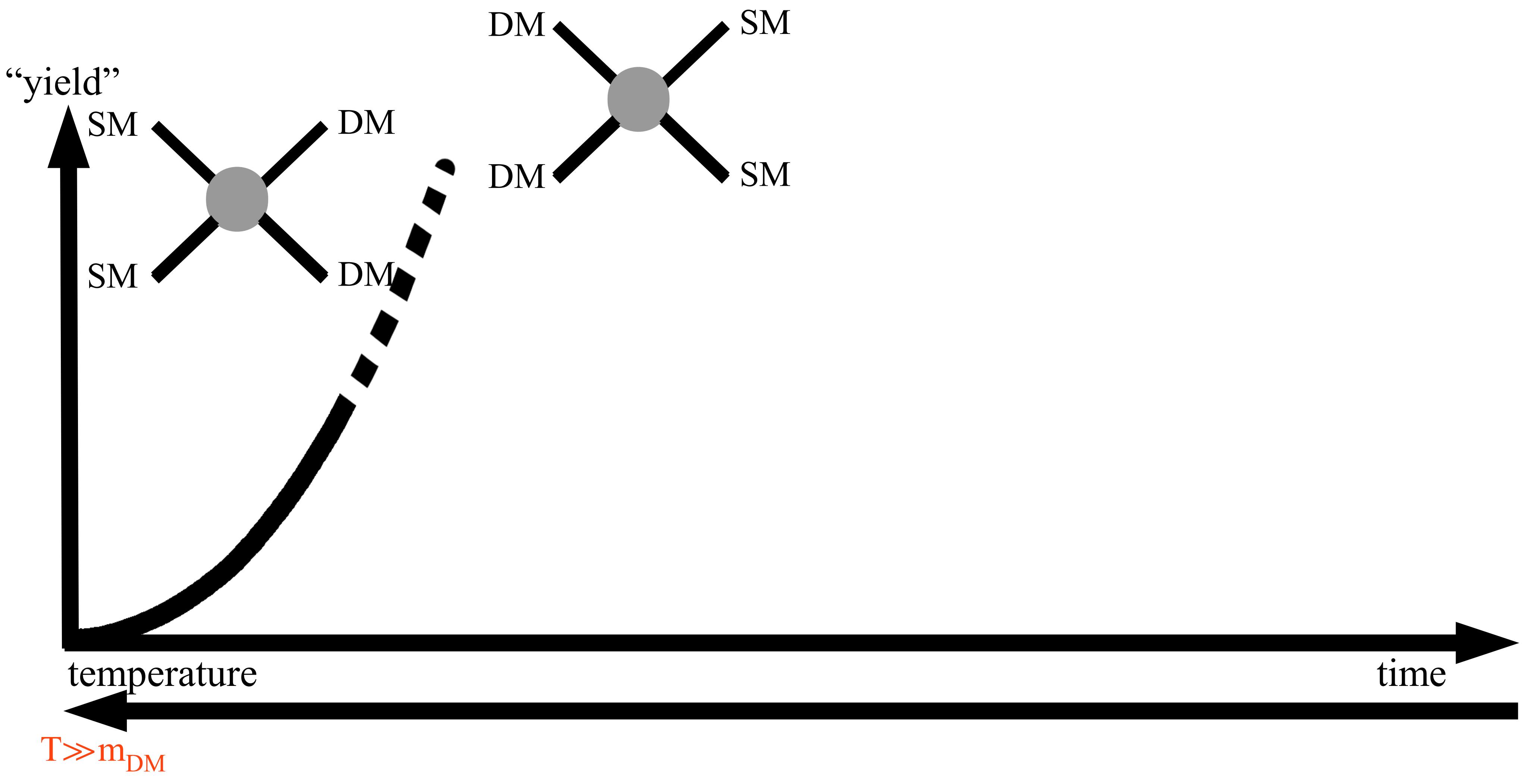


Assume that the temperature of the Universe after reheating was much larger than the DM mass.

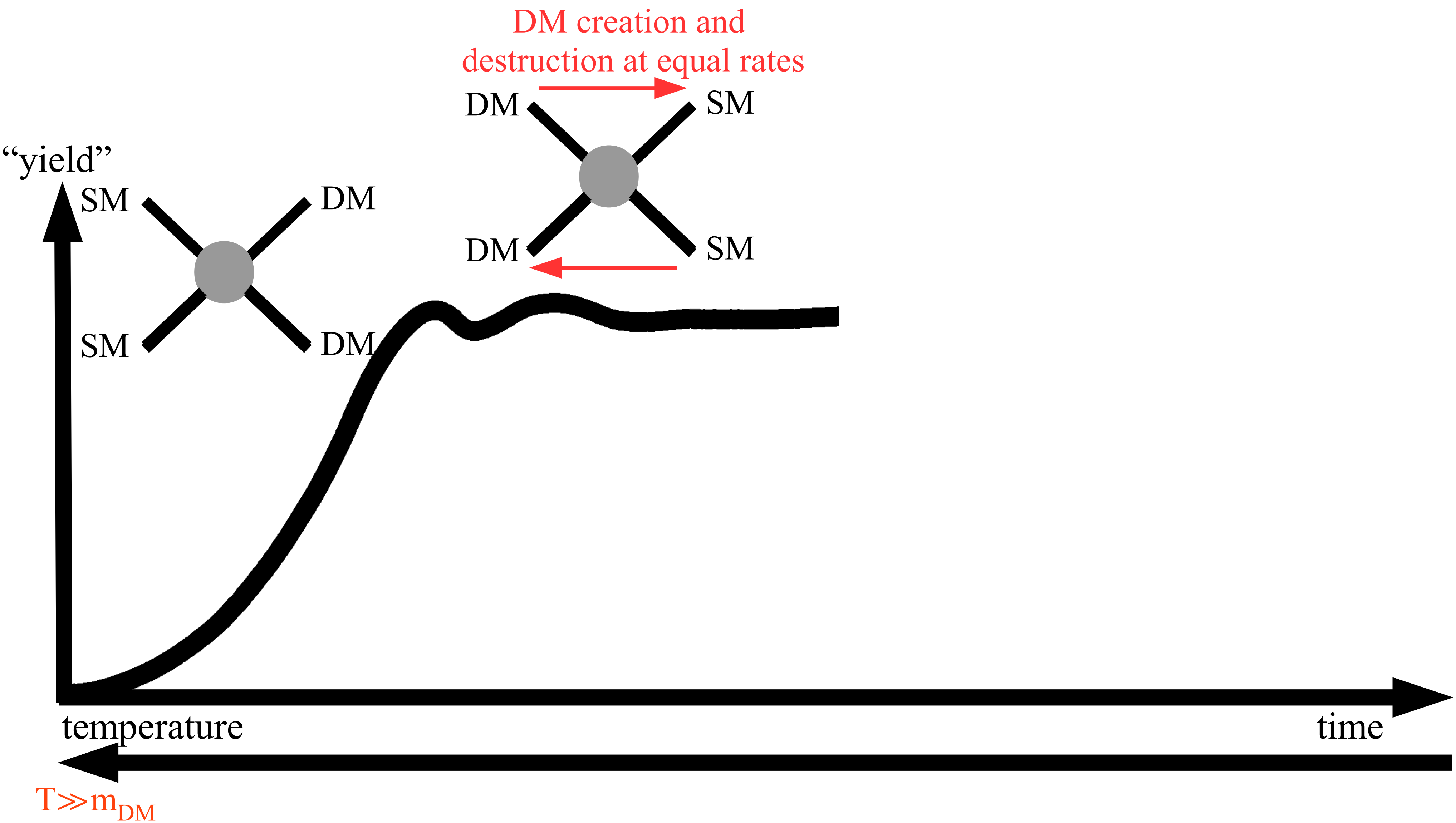
WIMP history (in a nutshell)



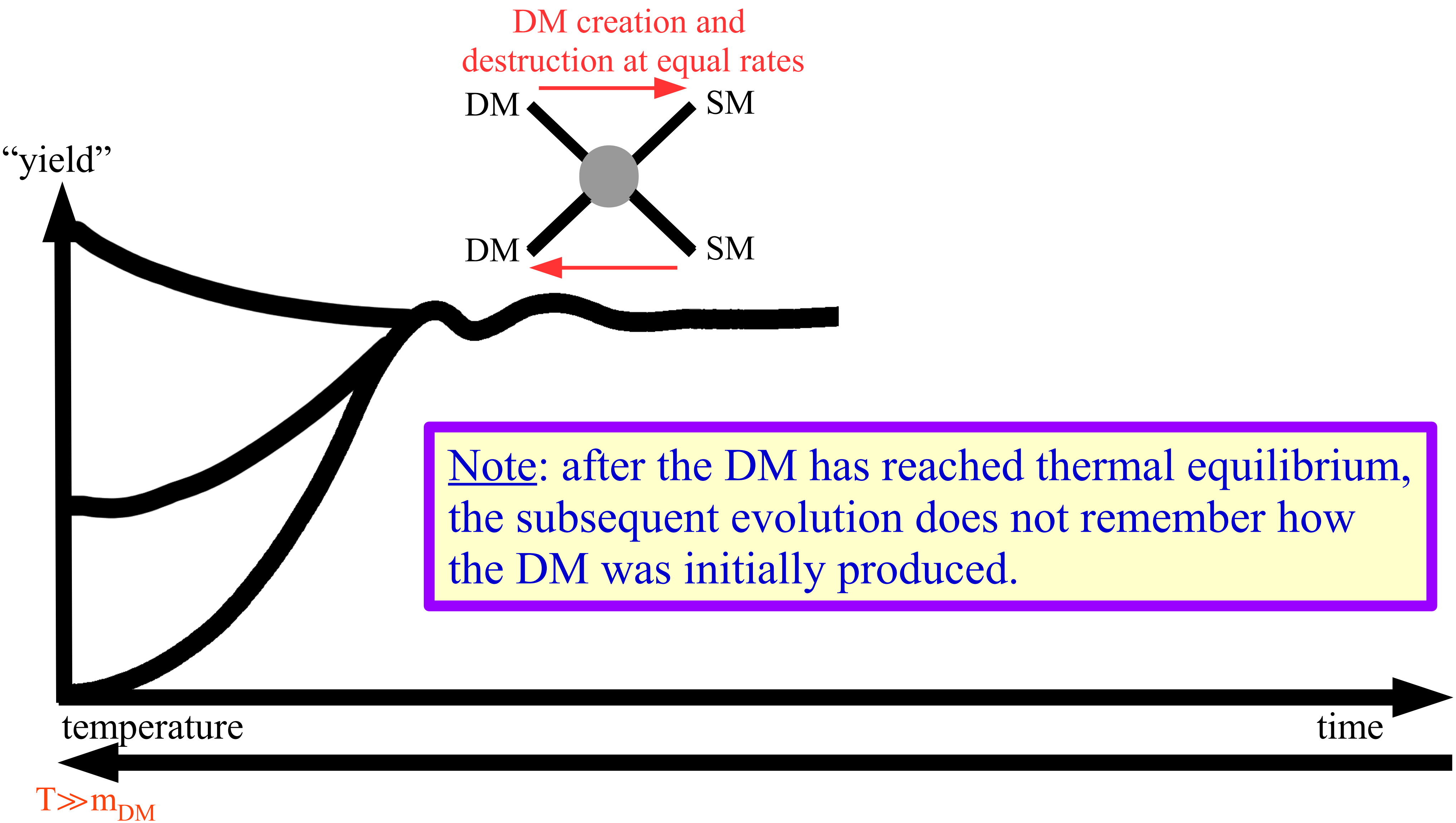
WIMP history (in a nutshell)



WIMP history (in a nutshell)

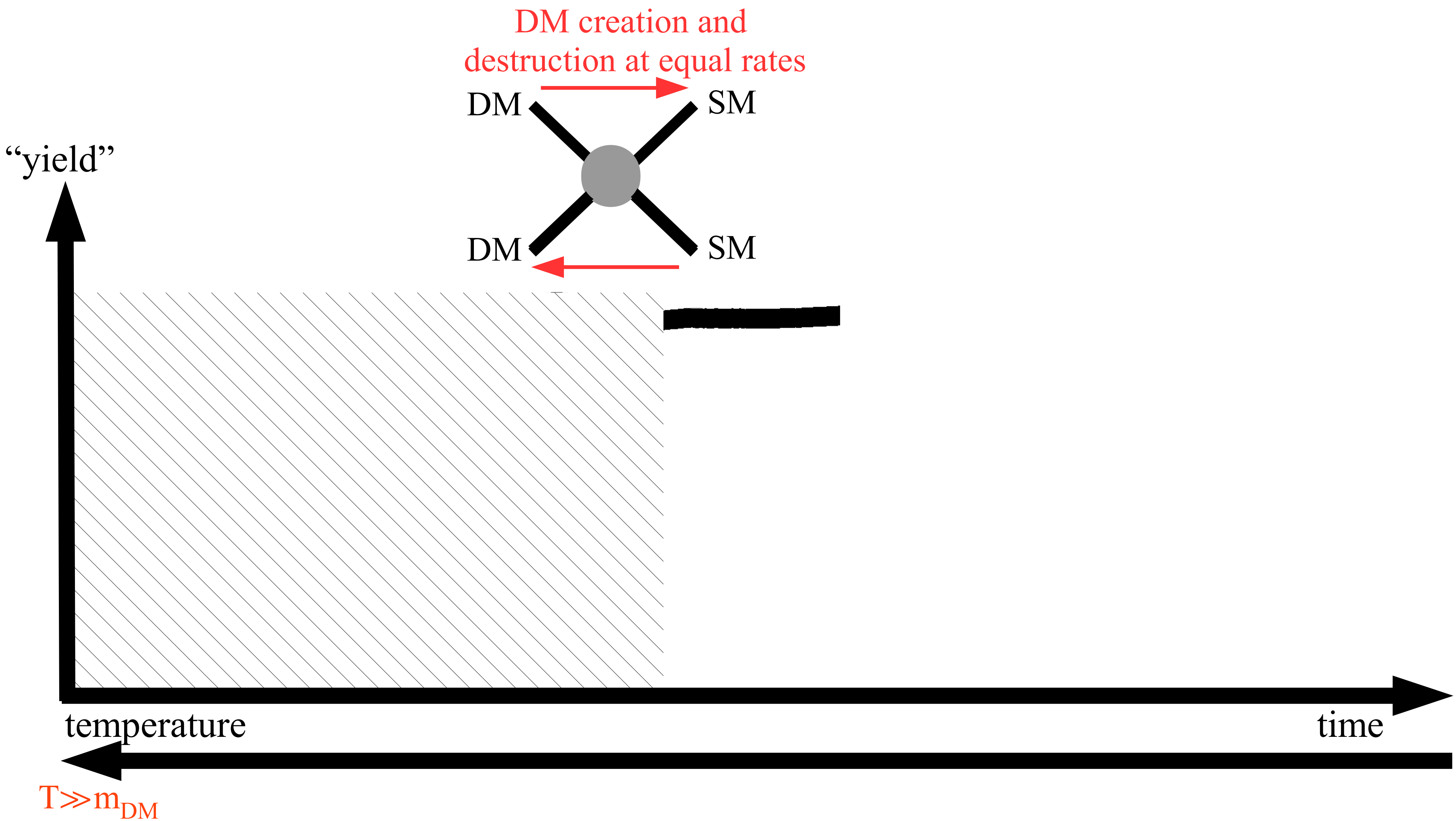


WIMP history (in a nutshell)

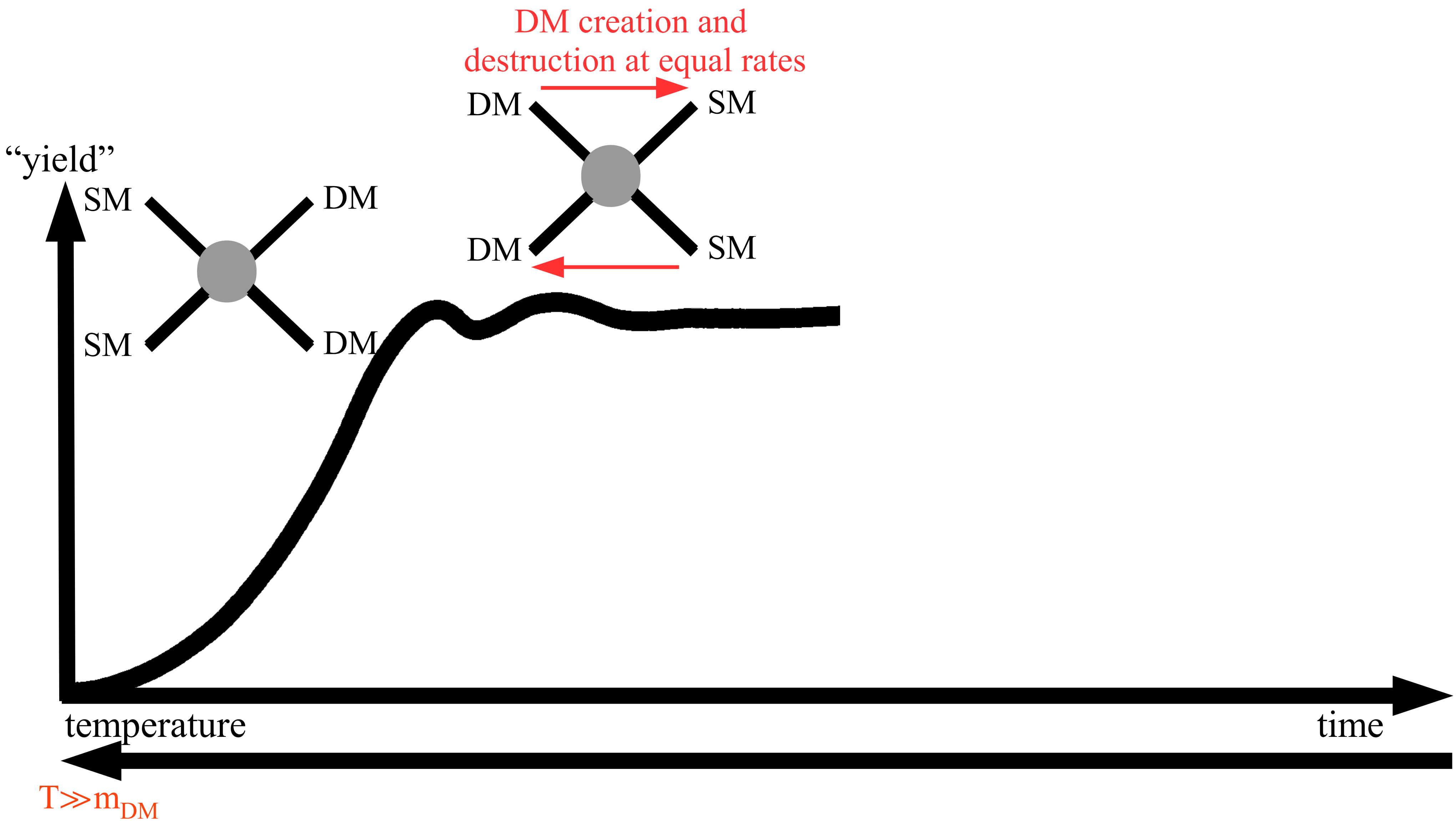


Note: after the DM has reached thermal equilibrium, the subsequent evolution does not remember how the DM was initially produced.

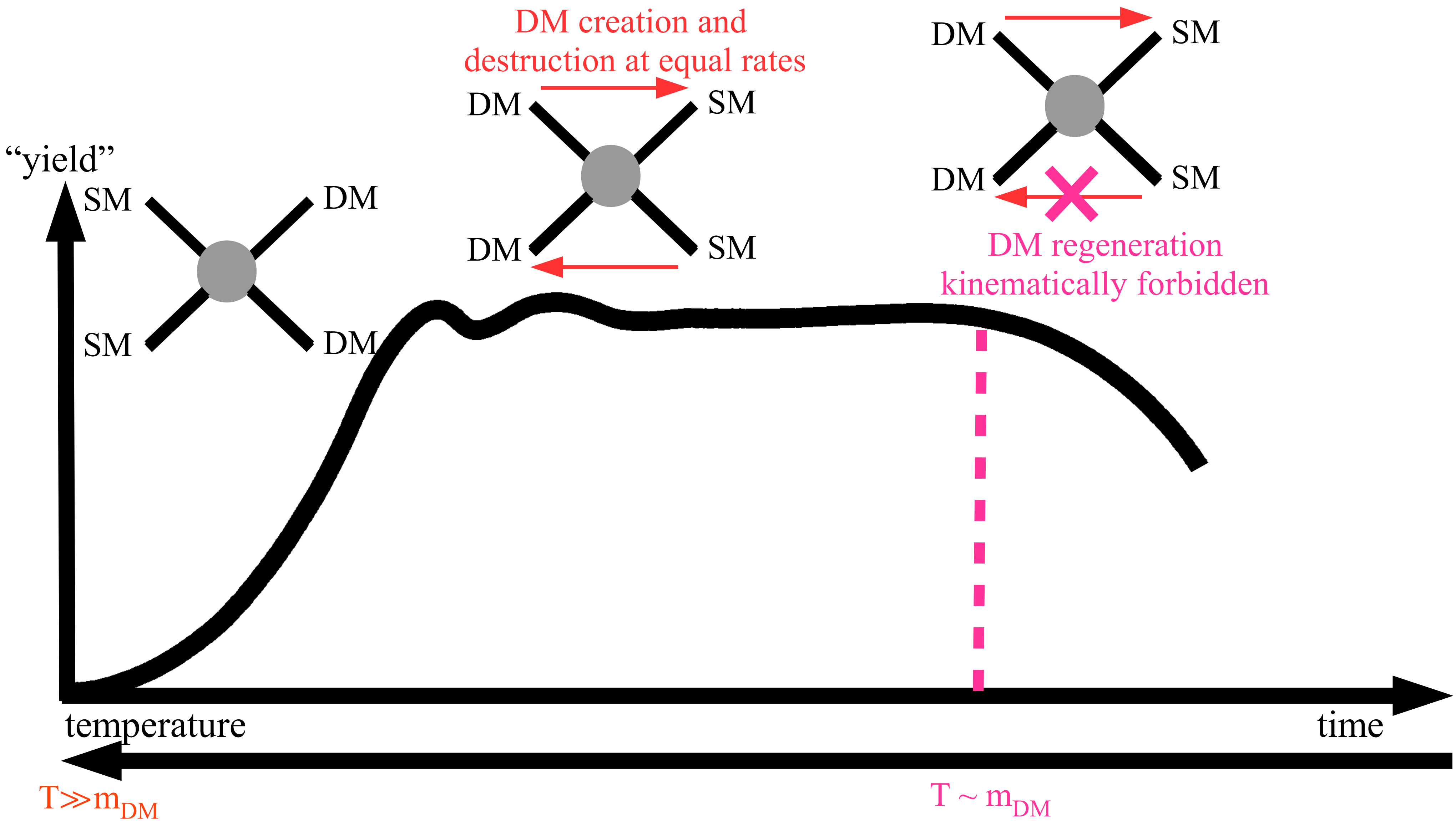
WIMP history (in a nutshell)



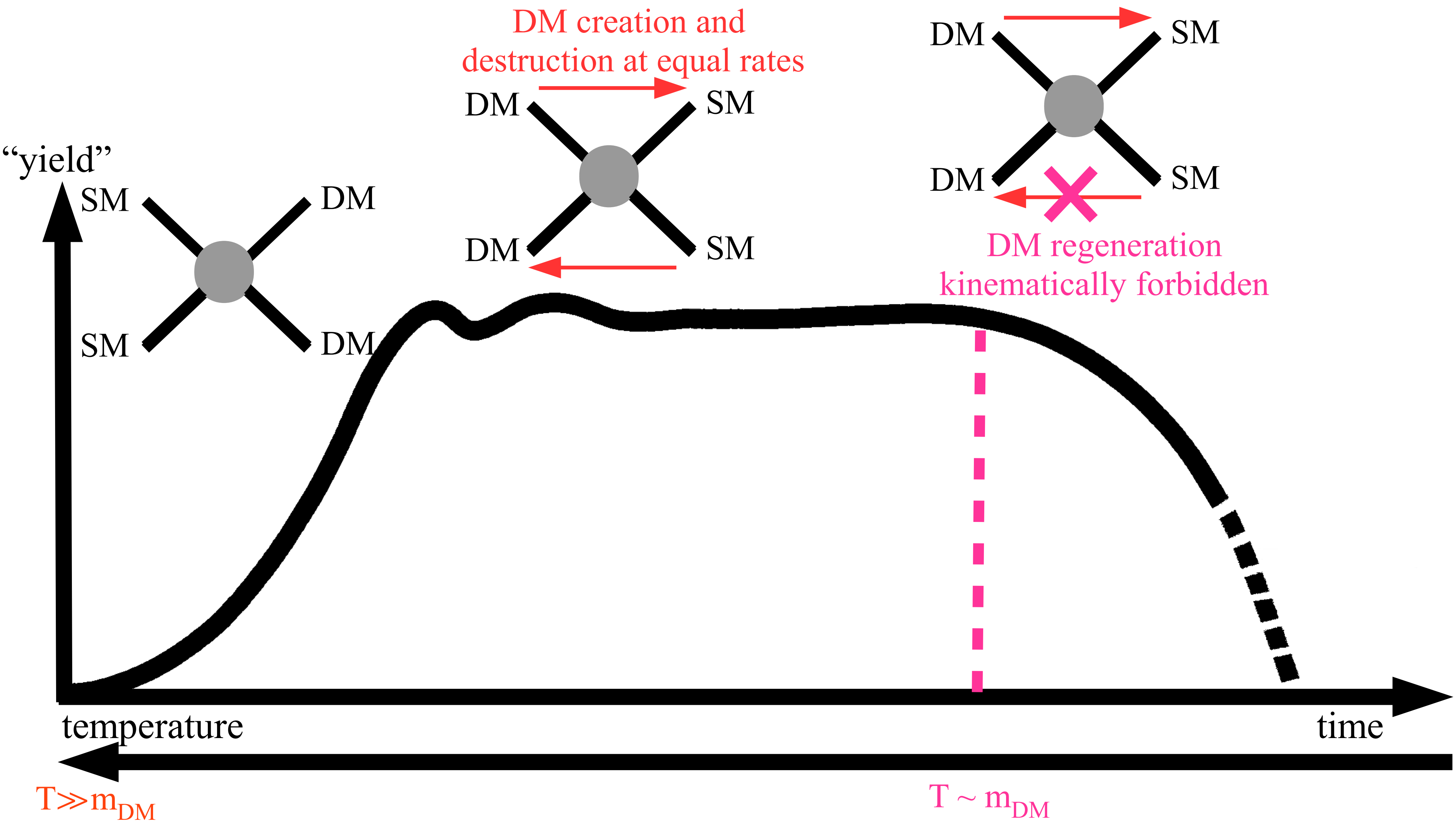
WIMP history (in a nutshell)



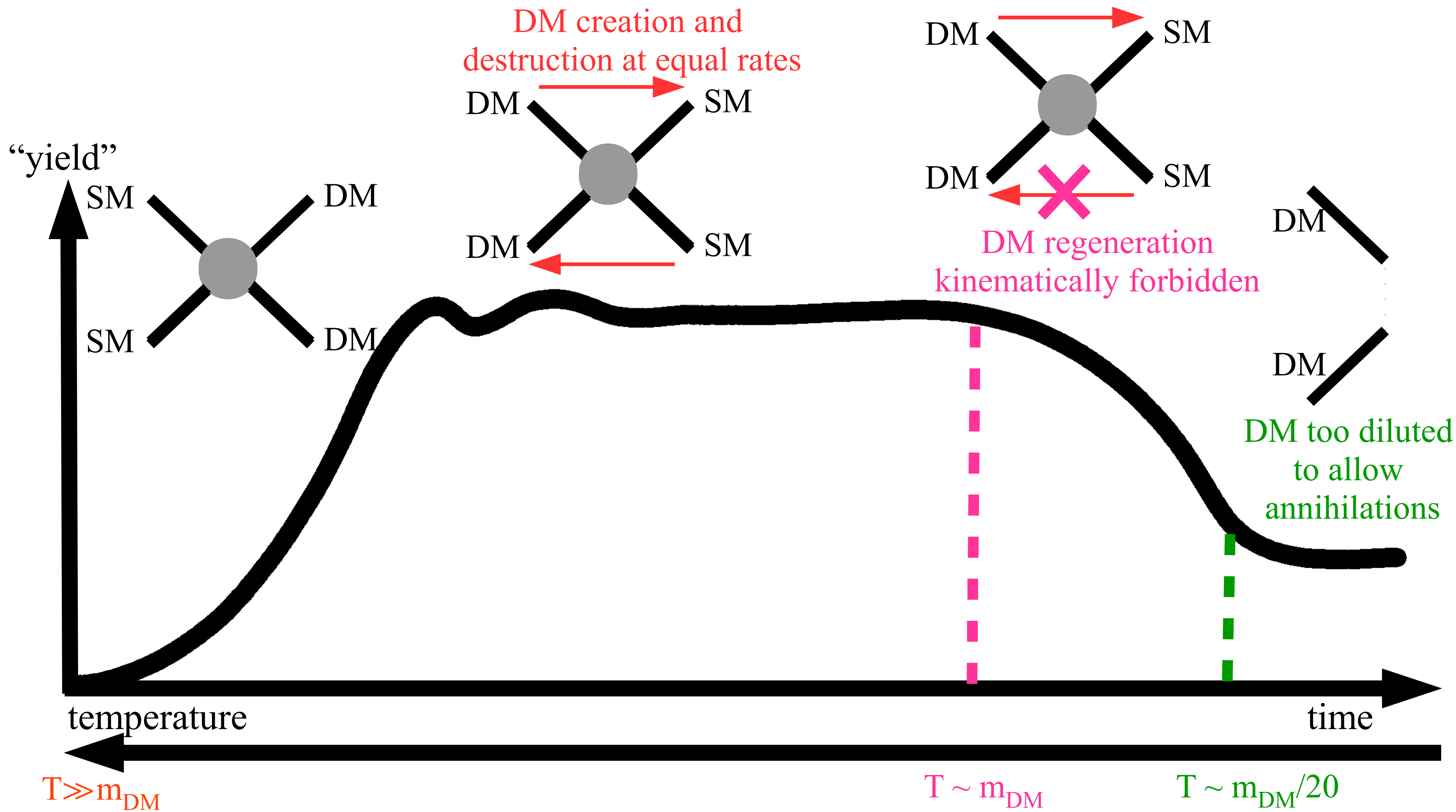
WIMP history (in a nutshell)



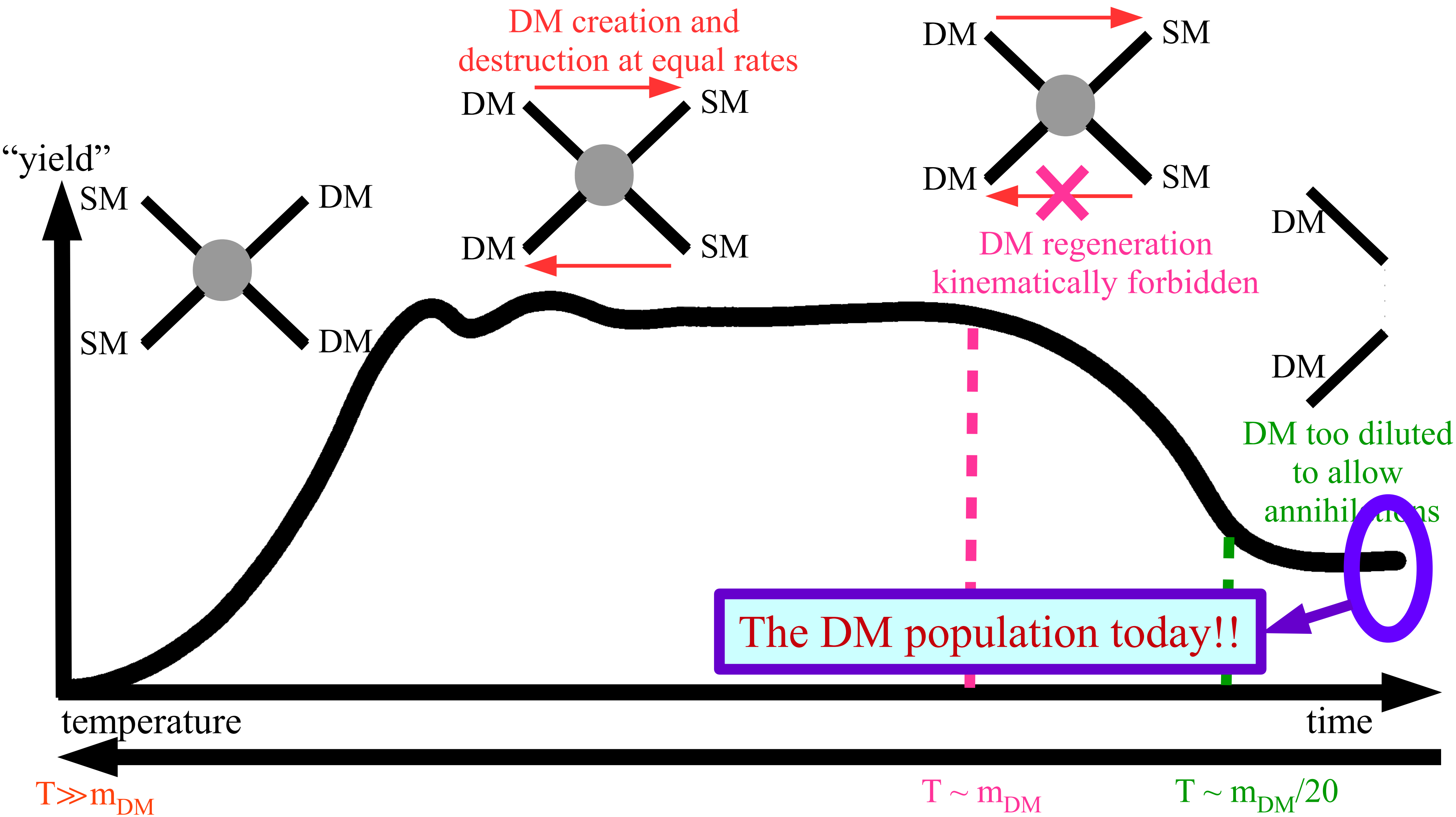
WIMP history (in a nutshell)



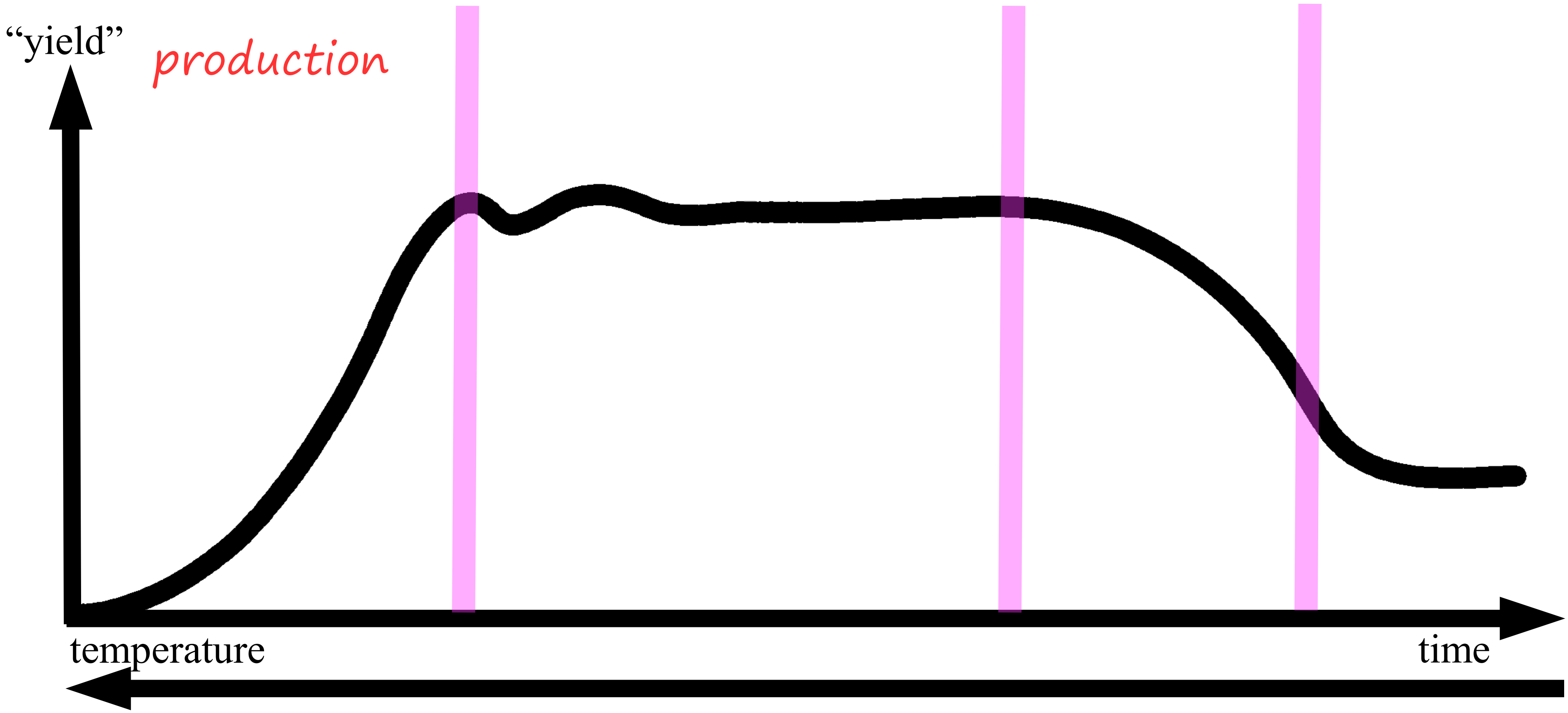
WIMP history (in a nutshell)



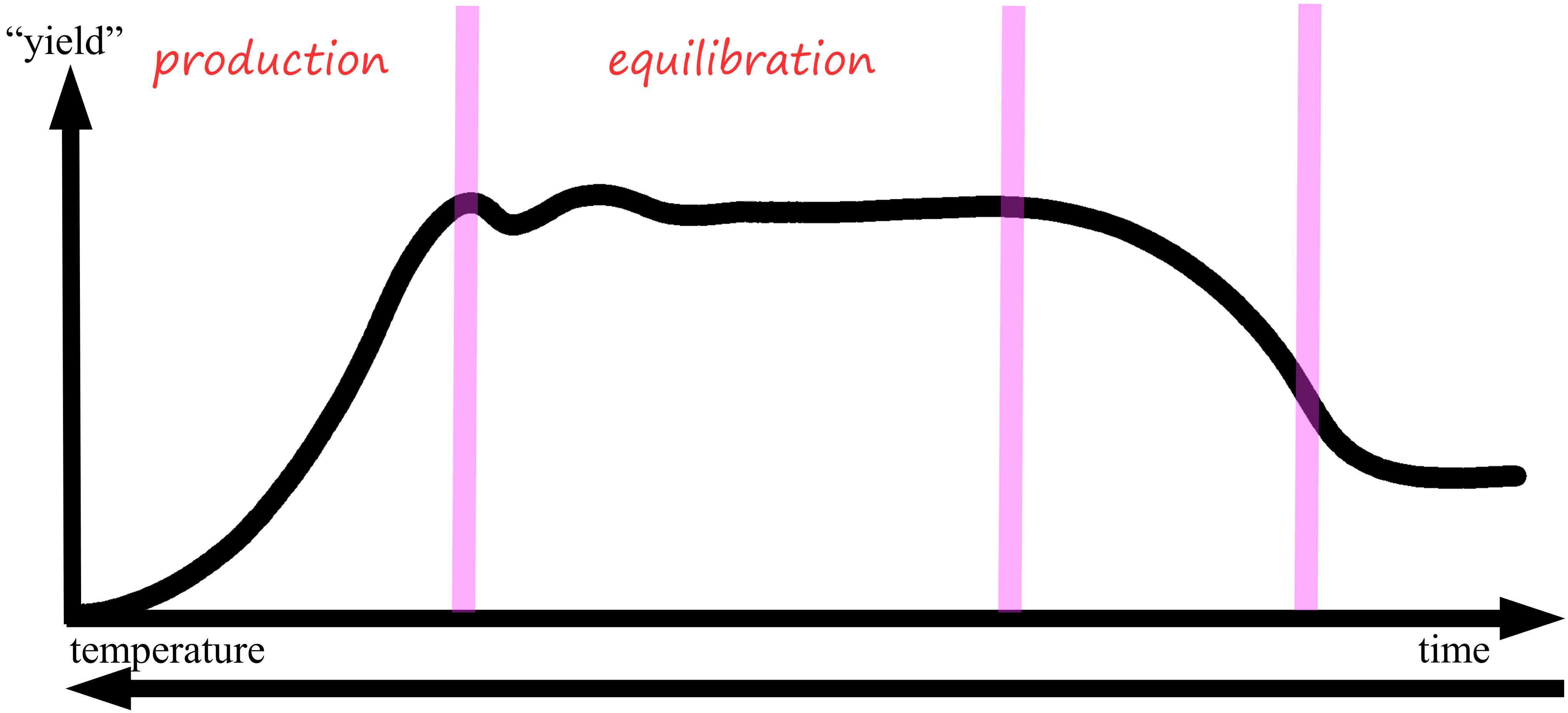
WIMP history (in a nutshell)



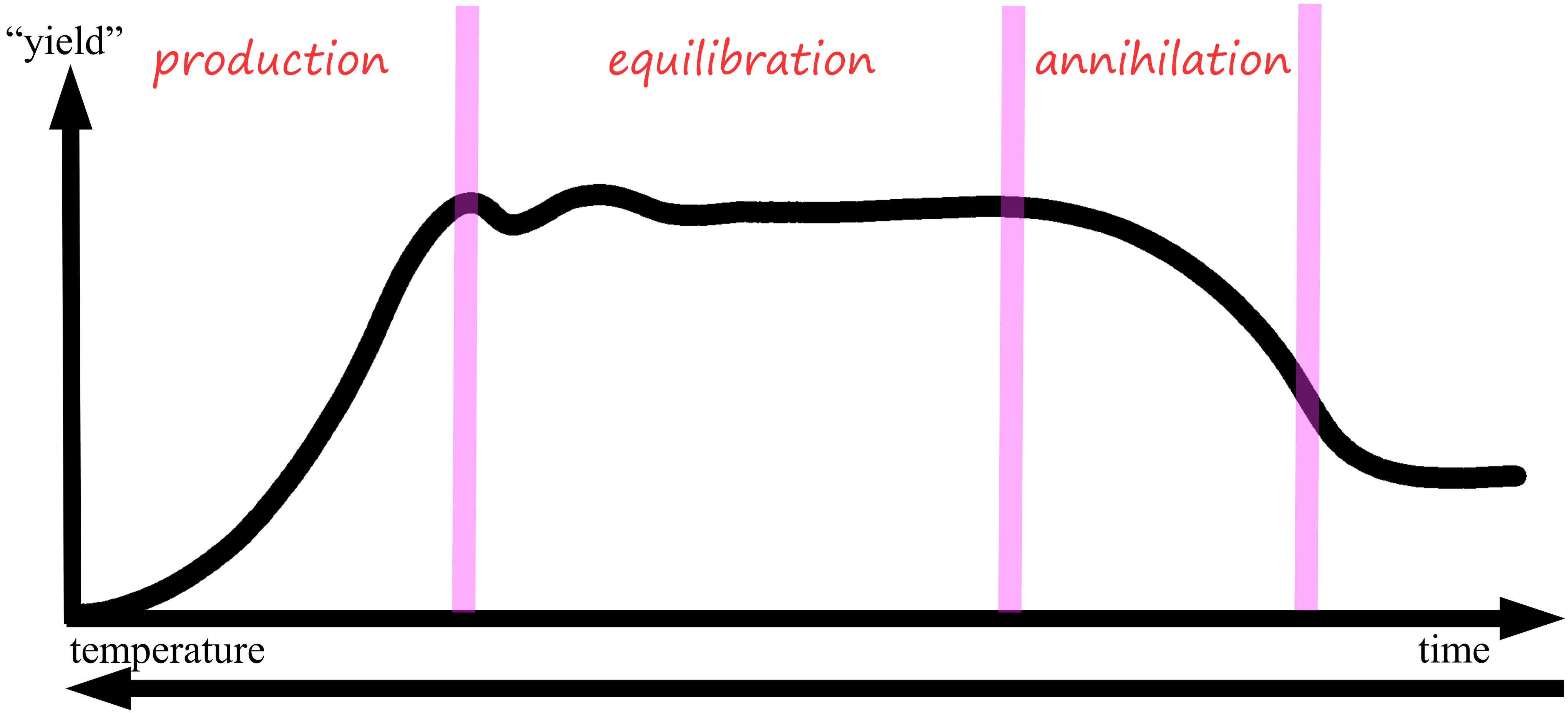
WIMP history (in a nutshell)



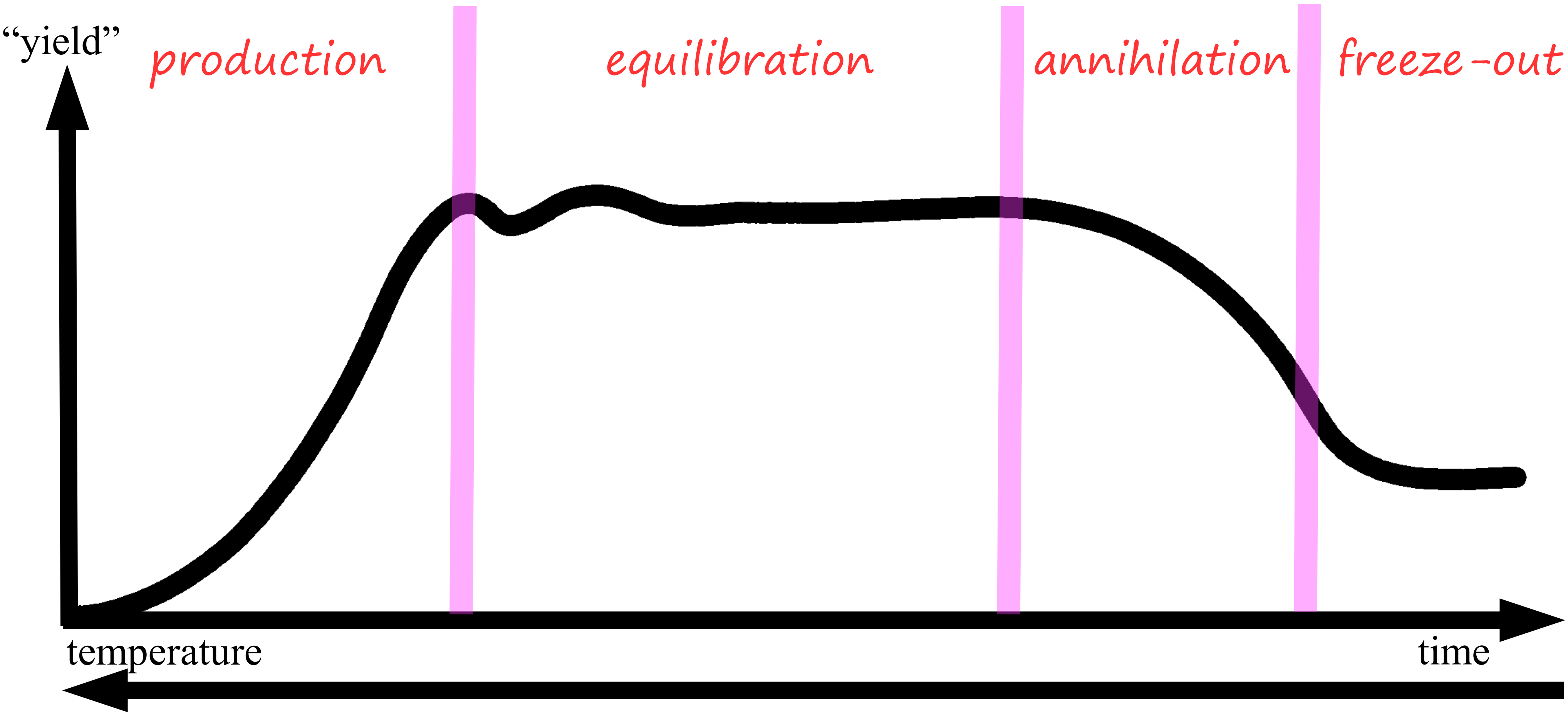
WIMP history (in a nutshell)



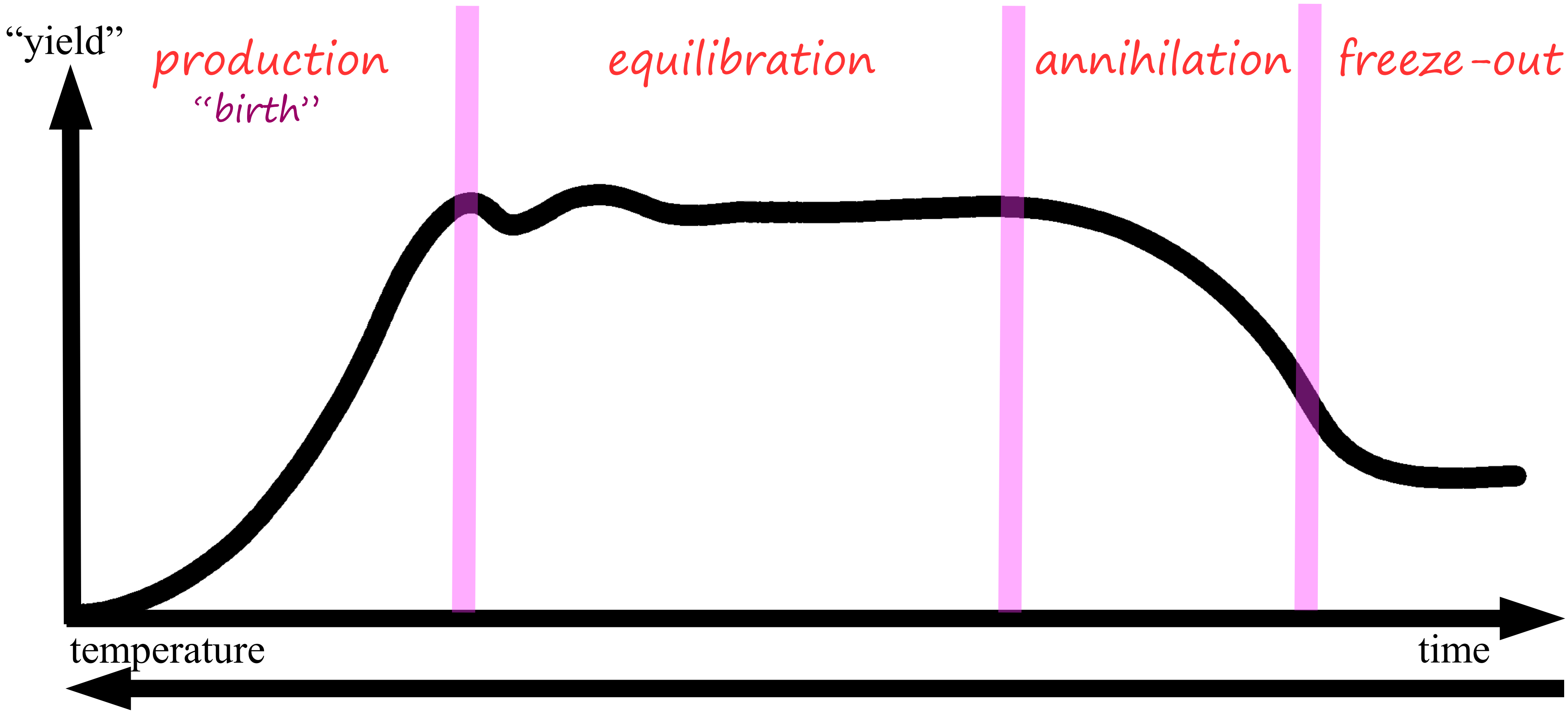
WIMP history (in a nutshell)



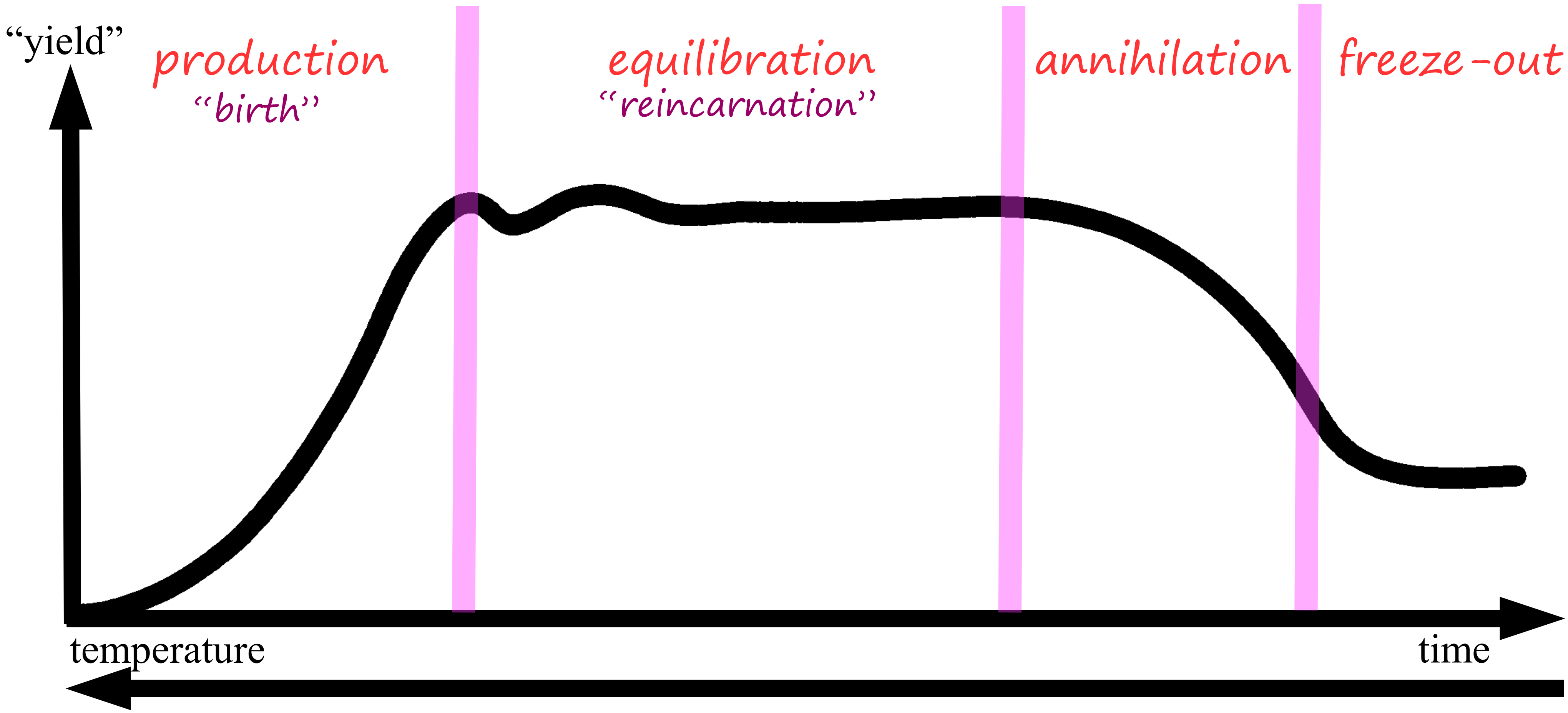
WIMP history (in a nutshell)



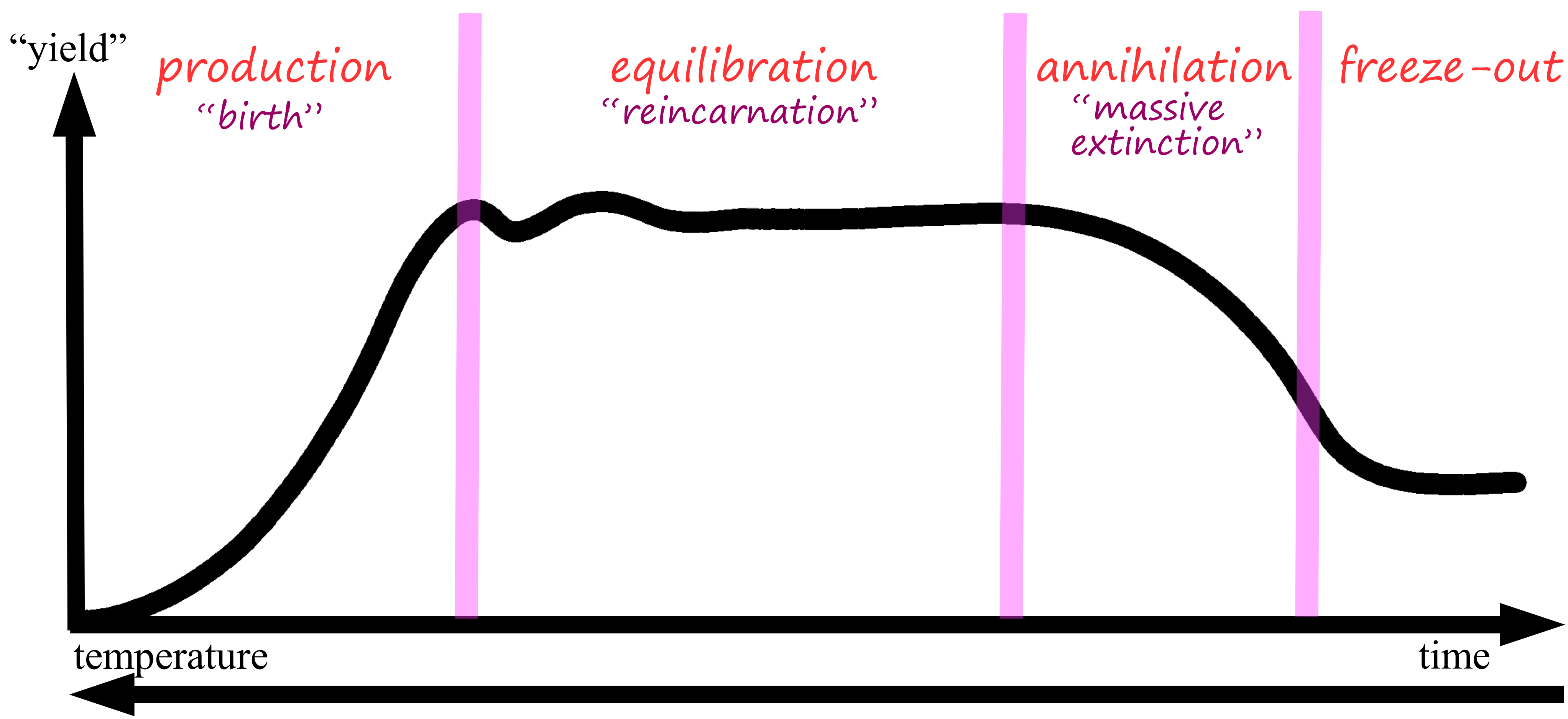
WIMP history (in a nutshell)



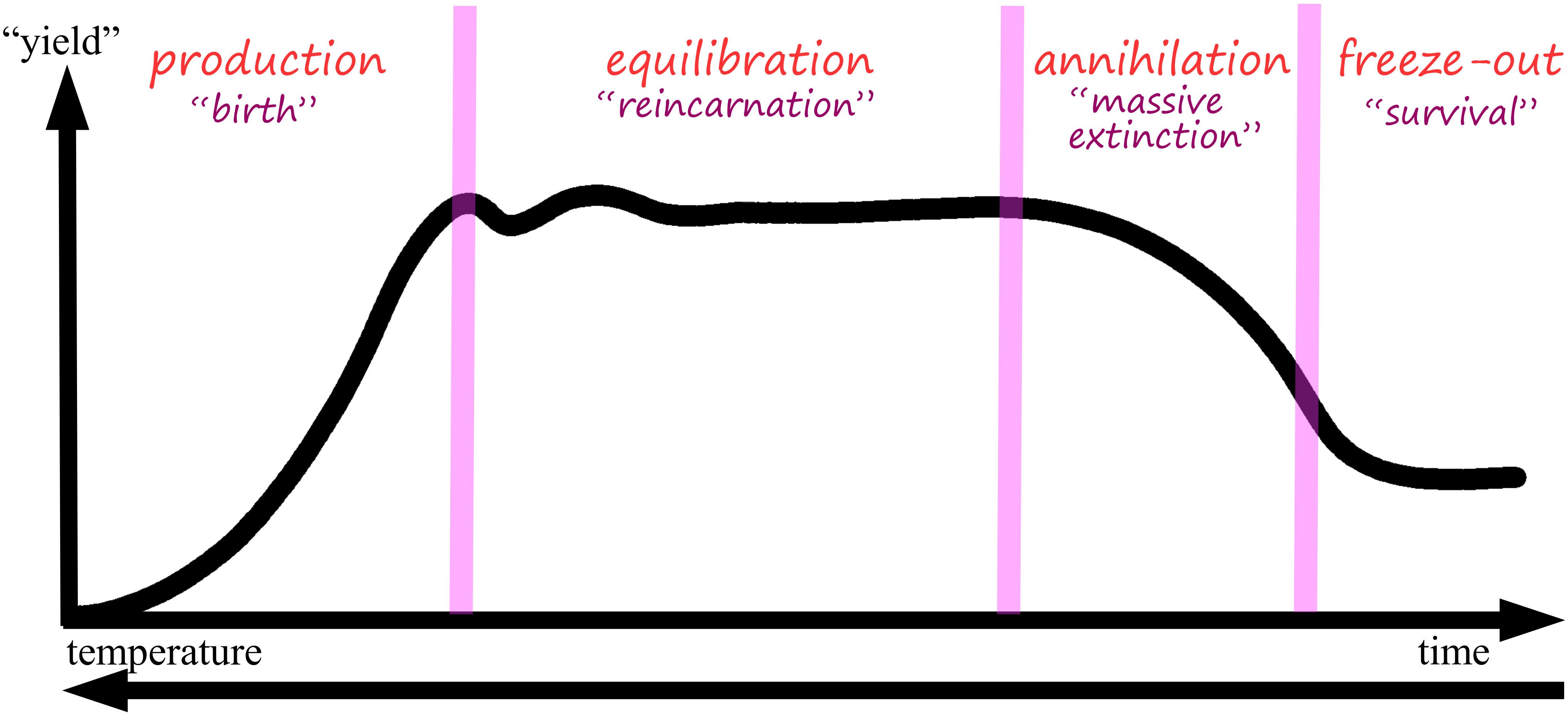
WIMP history (in a nutshell)



WIMP history (in a nutshell)



WIMP history (in a nutshell)



The basic tool: the Boltzmann equation

Boltzmann equation: equation that describes the time evolution of the phase space density distribution $f(t, \vec{r}, \vec{p})$:

$$\boxed{L[f] = C[f]}$$

Liouville operator
(time evolution)

Collision term
(creation/destruction of particles in phase space due to annihilations or decays)

The basic tool: the Boltzmann equation

- Liouville operator in classical mechanics

$$
\begin{aligned}
L[f] = \frac{df}{dt} &= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial p^i}\frac{dp^i}{dt} \\
&= \frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\cdot\vec{\nabla}_x f + \vec{F}\cdot\vec{\nabla}_p f
\end{aligned}
$$

The basic tool: the Boltzmann equation

- Liouville operator in classical mechanics

$$L[f] = \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial p^i}\frac{dp^i}{dt}$$
$$= \frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\cdot\vec{\nabla}_x f + \vec{F}\cdot\vec{\nabla}_p f$$

- Covariant Liouville operator

$$L[f] = \frac{df}{d\tau} = \frac{\partial f}{\partial x^\mu}\frac{dx^\mu}{d\tau} + \frac{\partial f}{\partial p^\mu}\frac{dp^\mu}{d\tau}$$

Geodesic equation

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau}\frac{dx^\sigma}{d\tau} = 0 \longrightarrow \frac{dp^\mu}{d\tau} = -\Gamma^\mu_{\rho\sigma}p^\rho p^\sigma$$

$$\rightsquigarrow L[f] = \frac{\partial f}{\partial x^\mu}p^\mu - \Gamma^\mu_{\rho\sigma}p^\rho p^\sigma\frac{\partial f}{\partial p^\mu}$$

The basic tool: the Boltzmann equation

Boltzmann equation for the dark matter number density in an expanding Universe:

$$\boxed{\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)}$$

(under some assumptions – see later)

The basic tool: the Boltzmann equation

Boltzmann equation for the dark matter number density in an expanding Universe:

$$\boxed{\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)}$$

(under some assumptions – see later)

- $n(t)$ number density of DM particles

$$n(t) = \frac{g}{(2\pi)^3} \int d^3p\, f(t, \vec{r}, \vec{p})$$

The basic tool: the Boltzmann equation

Boltzmann equation for the dark matter number density in an expanding Universe:

$$\boxed{\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)}$$

(under some assumptions – see later)

- $n(t)$ number density of DM particles

$$n(t) = \frac{g}{(2\pi)^3} \int d^3p \, f(t, \vec{r}, \vec{p})$$

$$\text{However} \begin{cases} \text{No } \vec{r} \text{ dependence (homogeneity)} \\ \text{No } \dfrac{\vec{p}}{|\vec{p}|} \text{ dependence (isotropy)} \end{cases}$$

The basic tool: the Boltzmann equation

Boltzmann equation for the dark matter number density in an expanding Universe:

$$\boxed{\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)}$$

(under some assumptions – see later)

- $n(t)$ number density of DM particles

$$n(t) = \frac{g}{(2\pi)^3} \int d^3p \, f(t, E)$$

$$\text{However} \begin{cases} \text{No } \vec{r} \text{ dependence (homogeneity)} \\ \text{No } \dfrac{\vec{p}}{|\vec{p}|} \text{ dependence (isotropy)} \end{cases}$$

The basic tool: the Boltzmann equation

Boltzmann equation for the dark matter number density in an expanding Universe:

$$\boxed{\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)}$$

(under some assumptions – see later)

- $n(t)$ number density of DM particles

$$n(t) = \frac{g}{(2\pi)^3} \int d^3p\, f(t, E)$$

However $\begin{cases} \text{No } \vec{r} \text{ dependence (homogeneity)} \\ \text{No } \dfrac{\vec{p}}{|\vec{p}|} \text{ dependence (isotropy)} \end{cases}$

- $H(t) \equiv \dfrac{\dot{a}}{a} \rightarrow$ Hubble rate

The basic tool: the Boltzmann equation

Boltzmann equation for the dark matter number density in an expanding Universe:

$$\boxed{\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)}$$

(under some assumptions – see later)

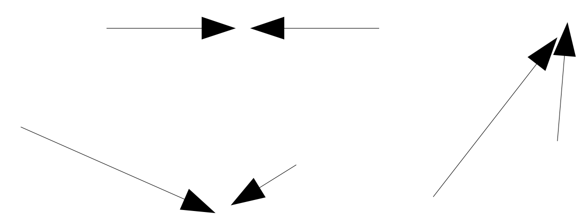
- $n(t)$ number density of DM particles

$$n(t) = \frac{g}{(2\pi)^3} \int d^3p\, f(t,E)$$

However $\begin{cases} \text{No } \vec{r} \text{ dependence (homogeneity)} \\ \text{No } \dfrac{\vec{p}}{|\vec{p}|} \text{ dependence (isotropy)} \end{cases}$

- $H(t) \equiv \dfrac{\dot{a}}{a}$ → Hubble rate

- σ = annihilation cross-section DM DM → SM SM
- v = relative velocity
- $\langle ... \rangle$ = thermal average



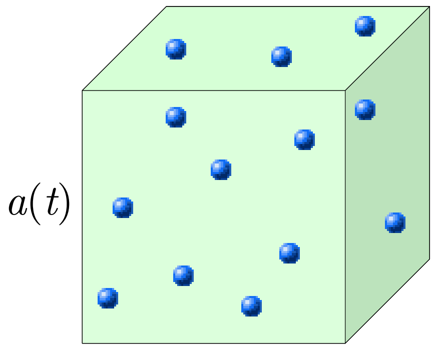
Justification

LHS Assume that the collision term vanishes (σ=0)
⇒ number of particles conserved

$$\frac{dN(t)}{dt} = 0$$

Justification

LHS Assume that the collision term vanishes (σ=0)
$\Rightarrow$ number of particles conserved



$$\frac{dN(t)}{dt} = 0$$

$$N(t) = n(t)V(t) = n(t)a(t)^3$$

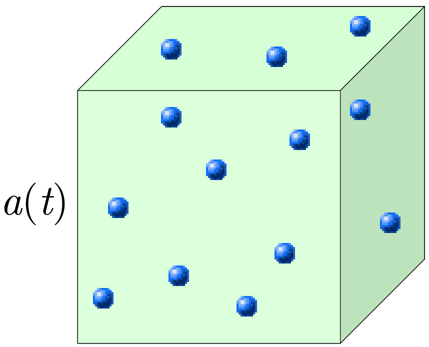
$$\rightsquigarrow \frac{d}{dt}(na^3) = \dot{n}a^3 + 3na^2\dot{a} = 0$$

$$\Rightarrow a^3(\dot{n} + 3\frac{\dot{a}}{a}n) = 0$$

Justification

LHS Assume that the collision term vanishes (σ=0)
$\Rightarrow$ number of particles conserved

$$\frac{dN(t)}{dt} = 0$$



$$N(t) = n(t)V(t) = n(t)a(t)^3$$

$$\rightsquigarrow \frac{d}{dt}(na^3) = \dot{n}a^3 + 3na^2\dot{a} = 0$$

$$\Rightarrow a^3(\dot{n} + 3\frac{\dot{a}}{a}n) = 0$$

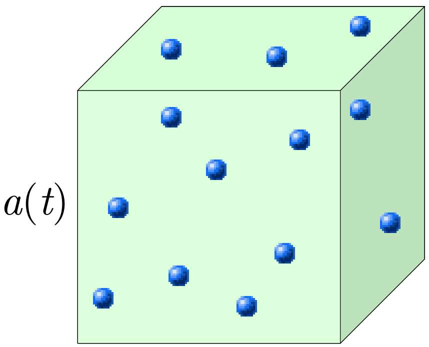
$$\boxed{\dot{n} + 3Hn = 0 \text{ when } \sigma = 0}$$

$$\left(\text{Full Boltzmann eq:} \quad \frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle(n^2 - n_{\text{eq}}^2)\right)$$

Justification

LHS Assume that the collision term vanishes (σ=0)
$\Rightarrow$ number of particles conserved

$$\frac{dN(t)}{dt} = 0$$



$$N(t) = n(t)V(t) = n(t)a(t)^3$$

$$\rightsquigarrow \frac{d}{dt}(na^3) = \dot{n}a^3 + 3na^2\dot{a} = 0$$

$$\Rightarrow a^3(\dot{n} + 3\frac{\dot{a}}{a}n) = 0$$

$$\boxed{\dot{n} + \underbrace{3Hn} = 0 \text{ when } \sigma = 0}$$

Dilution term due to the expansion of the Universe

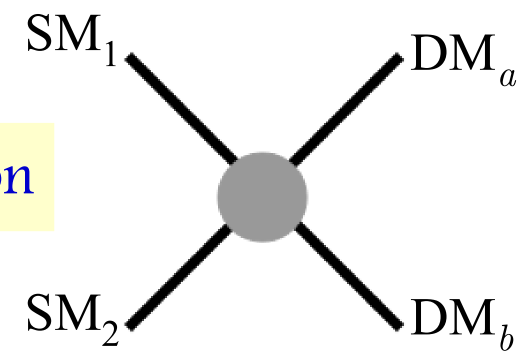
Justification

RHS

Remember:

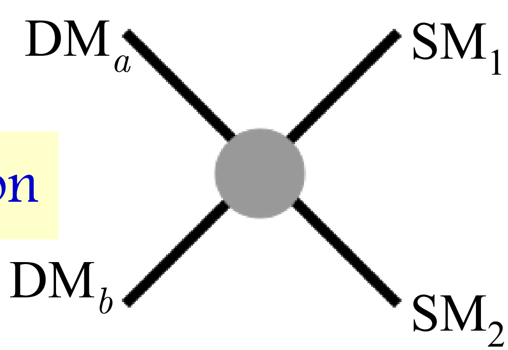
Boltzmann equation: change of n = production - destruction

production



$$\sim \int |\mathcal{M}_{12\to ab}|^2 \, f_1 \, f_2 \, d(\text{phase space})$$

destruction



$$\sim \int |\mathcal{M}_{ab\to 12}|^2 \, f_a \, f_b \, d(\text{phase space})$$

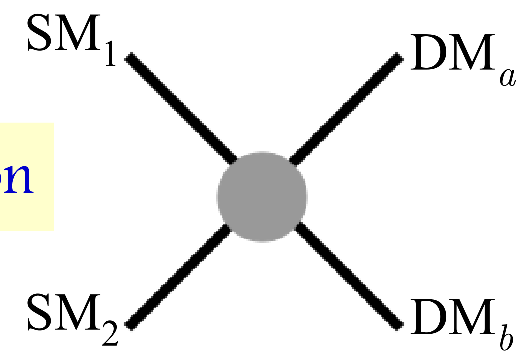
Justification

RHS

Remember:

Boltzmann equation: change of n = production - destruction

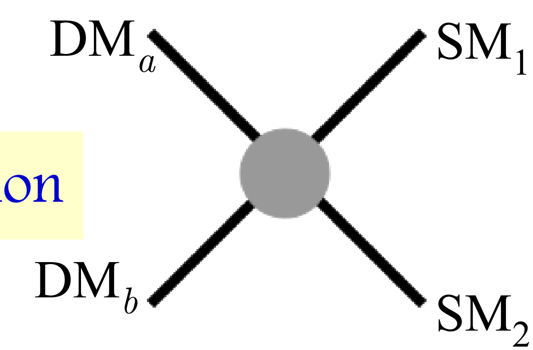
production



$$\sim \int |\mathcal{M}_{12\to ab}|^2\, f_1\, f_2\, d(\text{phase space})$$

Equal if T conserved
(CP conserved)

destruction



$$\sim \int |\mathcal{M}_{ab\to 12}|^2\, f_a\, f_b\, d(\text{phase space})$$

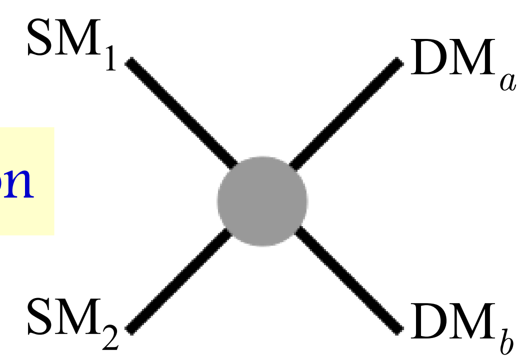
Justification

RHS

Remember:

Boltzmann equation: change of n = production - destruction

production

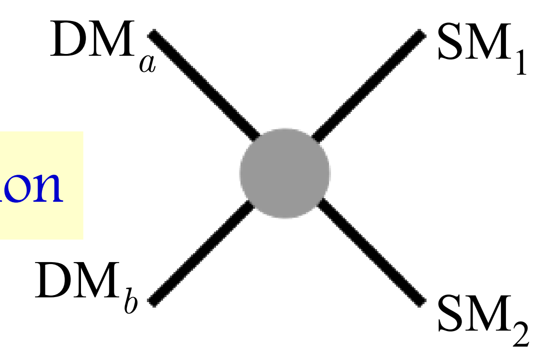


$$\sim \int |\mathcal{M}_{12\to ab}|^2 \, f_1 \, f_2 \, d(\text{phase space})$$

Equal if T conserved
(CP conserved)

Assumption 1

destruction



$$\sim \int |\mathcal{M}_{ab\to 12}|^2 \, f_a \, f_b \, d(\text{phase space})$$

$$\boxed{\text{RHS} \sim -\int |\mathcal{M}_{ab\to 12}|^2 \, (f_a \, f_b - f_1 \, f_2) \, d(\text{phase space})}$$

Justification

RHS Assume SM particles in thermal equilibrium **Assumption 2**

$$f_1 = f_1^{\mathrm{eq}} = e^{-E_1/T} \quad \text{(Boltzmann distribution)}$$

Justification

RHS Assume SM particles in thermal equilibrium **Assumption 2**

$$f_1 = f_1^{\rm eq} = e^{-E_1/T} \quad \text{(Boltzmann distribution)}$$

$$\Rightarrow f_1\, f_2 = f_1^{\rm eq} f_2^{\rm eq} = e^{-(E_1+E_2)/T} = e^{-(E_a+E_b)/T} = f_a^{\rm eq} f_b^{\rm eq}$$

$$\Rightarrow \text{RHS} \sim -\int |\mathcal{M}_{ab\to 12}|^2 \,(f_a\, f_b - f_a^{\rm eq} f_b^{\rm eq})\, d(\text{phase space})$$

Justification

RHS Assume SM particles in thermal equilibrium *Assumption 2*

$$f_1 = f_1^{\rm eq} = e^{-E_1/T} \quad \text{(Boltzmann distribution)}$$

$$\Rightarrow f_1\, f_2 = f_1^{\rm eq} f_2^{\rm eq} = e^{-(E_1+E_2)/T} = e^{-(E_a+E_b)/T} = f_a^{\rm eq} f_b^{\rm eq}$$

$$\Rightarrow \text{RHS} \sim -\int |\mathcal{M}_{ab\to 12}|^2 \,(f_a\, f_b - f_a^{\rm eq} f_b^{\rm eq})\, d(\text{phase space})$$

$$n(t) = \frac{g}{(2\pi)^3}\int d^3p\, f(E,t)$$

$$\sigma = \frac{1}{\text{flux}}\int |\mathcal{M}|^2$$

$$\boxed{\text{RHS} = -\langle\sigma v\rangle (n_a n_b - n_a^{\rm eq} n_b^{\rm eq})}$$

$$\left(\text{Full Boltzmann eq:} \quad \frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle (n^2 - n_{\rm eq}^2)\right)$$

Solving the Boltzmann equation

Boltzmann equation:

$$\frac{dn}{dt} + \underbrace{3Hn} = -\langle \sigma v \rangle (n^2 - n_{\mathrm{eq}}^2)$$

Number density reduced by the expansion of the Universe.
Hot to tell whether the dark matter production/destruction is efficient or not?

Solving the Boltzmann equation

Boltzmann equation:

$$\frac{dn}{dt} + \underbrace{3Hn}_{} = -\langle\sigma v\rangle(n^2 - n_{\rm eq}^2)$$

Number density reduced by the expansion of the Universe.
Hot to tell whether the dark matter production/destruction is efficient or not?

Define "yield": $\boxed{Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}}$

If no entropy production, $$\frac{dS}{dt} = 0 = \frac{d}{dt}(a^3 s) = a^3(\dot{s} + 3Hs)$$

$$\rightsquigarrow \dot{s} + 3Hs = 0$$

Solving the Boltzmann equation

Boltzmann equation:

$$\frac{dn}{dt} + \underbrace{3Hn} = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)$$

Number density reduced by the expansion of the Universe.
Hot to tell whether the dark matter production/destruction is efficient or not?

Define "yield": $$\boxed{Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}}$$

Time evolution of the yield:

$$\frac{dY}{dt} = \frac{1}{s^2}(\dot{n}s - n\dot{s}) = \frac{1}{s^2}(-3Hns - s\langle \sigma v \rangle (n^2 - n_{\rm eq}^2) + n3HS)$$

$$\dot{s} = -3Hs$$

$$\dot{n} = -3Hn - \langle \sigma v \rangle (n^2 - n_{\rm eq}^2)$$

Solving the Boltzmann equation

Boltzmann equation:

$$\frac{dn}{dt} + \underbrace{3Hn}_{} = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)$$

Number density reduced by the expansion of the Universe.
Hot to tell whether the dark matter production/destruction is efficient or not?

Define "yield": $$\boxed{Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}}$$

Time evolution of the yield:

$$\frac{dY}{dt} = \frac{1}{s^2}(\dot{n}s - n\dot{s}) = \frac{1}{s^2}(\cancel{-3Hns} - s\langle \sigma v \rangle (n^2 - n_{\rm eq}^2) + \cancel{n3HS})$$

$$\dot{s} = -3Hs$$

$$\dot{n} = -3Hn - \langle \sigma v \rangle (n^2 - n_{\rm eq}^2)$$

Solving the Boltzmann equation

Boltzmann equation:

$$\frac{dn}{dt} + \underbrace{3Hn}_{} = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)$$

Number density reduced by the expansion of the Universe.
Hot to tell whether the dark matter production/destruction is efficient or not?

Define “yield”: $$\boxed{Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}}$$

Time evolution of the yield:

$$\boxed{\frac{dY}{dt} = -s\langle \sigma v \rangle (Y^2 - Y_{\rm eq}^2)}$$

$$Y_{\rm eq} \equiv \frac{n_{\rm eq}(t)}{s(t)}$$

Solving the Boltzmann equation

Boltzmann equation:

$$\frac{dn}{dt} + \underbrace{3Hn} = -\langle \sigma v \rangle (n^2 - n_{\rm eq}^2)$$

Number density reduced by the expansion of the Universe.
Hot to tell whether the dark matter production/destruction is efficient or not?

Define “yield”: $$\boxed{Y \equiv \frac{n}{s} = \frac{\text{number density}}{\text{entropy density}}}$$

Time evolution of the yield:

$$\boxed{\frac{dY}{dt} = -s\langle \sigma v \rangle (Y^2 - Y_{\rm eq}^2)}$$

If σ=0, then Y = constant

What is Y_{eq}

Equilibrium number density:

$$n_{\mathrm{eq}} = \frac{g}{(2\pi)^3} \int d^3p \, \underbrace{f_{\mathrm{eq}}(t, E)}_{e^{-E/T}} = \begin{cases} \sim T^3 & \text{if } T \gg m_{\mathrm{DM}} \\ \sim (m_{\mathrm{DM}} T)^{3/2} e^{-m_{\mathrm{DM}}/T} & \text{if } T \ll m_{\mathrm{DM}} \end{cases}$$

What is Y_{eq}

Equilibrium number density:

$$n_{\text{eq}} = \frac{g}{(2\pi)^3} \int d^3p \, \underbrace{f_{\text{eq}}(t, E)}_{e^{-E/T}} = \begin{cases} \sim T^3 & \text{if } T \gg m_{\text{DM}} \\ \sim (m_{\text{DM}} T)^{3/2} e^{-m_{\text{DM}}/T} & \text{if } T \ll m_{\text{DM}} \end{cases}$$

Entropy density: $s \sim T^3$

$$\Rightarrow \boxed{Y_{\text{eq}} = \begin{cases} \sim \text{constant} & \text{if } T \gg m_{\text{DM}} \\ \sim \left(\frac{m_{\text{DM}}}{T}\right)^{3/2} e^{-m_{\text{DM}}/T} & \text{if } T \ll m_{\text{DM}} \end{cases}}$$

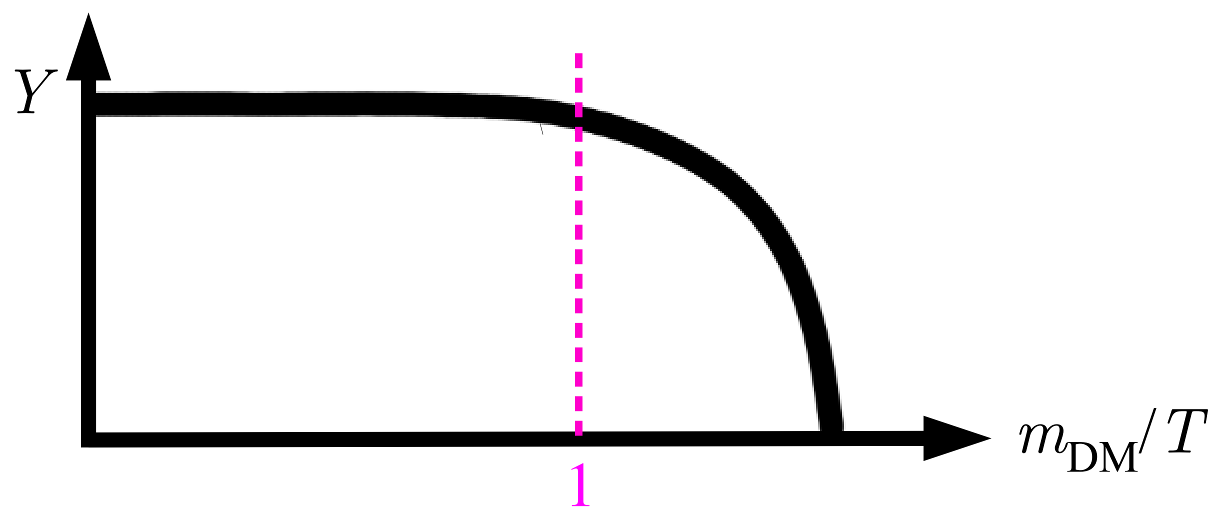
What is Y_{eq}

Equilibrium number density:

$$n_{\text{eq}} = \frac{g}{(2\pi)^3} \int d^3p \, \underbrace{f_{\text{eq}}(t,E)}_{e^{-E/T}} = \begin{cases} \sim T^3 & \text{if } T \gg m_{\text{DM}} \\ \sim (m_{\text{DM}}T)^{3/2} e^{-m_{\text{DM}}/T} & \text{if } T \ll m_{\text{DM}} \end{cases}$$

Entropy density: $s \sim T^3$

$$\Rightarrow \boxed{Y_{\text{eq}} = \begin{cases} \sim \text{constant} & \text{if } T \gg m_{\text{DM}} \\ \sim \left(\frac{m_{\text{DM}}}{T}\right)^{3/2} e^{-m_{\text{DM}}/T} & \text{if } T \ll m_{\text{DM}} \end{cases}}$$



Some tricks

1) Instead of the cosmic time, use as variable $x = \dfrac{m_{\rm DM}}{T}$

$$\rightsquigarrow \quad \frac{dY}{dt} = \frac{dY}{da}\,\frac{da}{dt}$$

Since $a \sim \dfrac{1}{T} \sim x$

$$\rightsquigarrow \quad \frac{dY}{dt} = \frac{dY}{dx}\,\frac{x}{a}\,\frac{da}{dt} = \frac{dY}{dx}\,x\,H$$

Therefore, $$\frac{dY}{dx} = -\frac{\langle\sigma v\rangle s}{H\,x}\left[Y^2 - Y_{\rm eq}^2\right]$$

Some tricks

2)Take $Y_{\rm eq}^2$ out of the bracket

$$\frac{dY}{dx} = -\frac{\langle \sigma v \rangle}{H\,x} s Y_{\rm eq} Y_{\rm eq}\Big[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\Big]$$

Some tricks

2)Take Y_{eq}^2 out of the bracket

$$\frac{dY}{dx} = -\frac{\langle\sigma v\rangle}{H\,x}\underbrace{sY_{\text{eq}}}_{n_{\text{eq}}}Y_{\text{eq}}\left[\left(\frac{Y}{Y_{\text{eq}}}\right)^2 - 1\right]$$

$$\Rightarrow \frac{x}{Y_{\text{eq}}}\frac{dY}{dx} = -\frac{n_{\text{eq}}\langle\sigma v\rangle}{H}\left[\left(\frac{Y}{Y_{\text{eq}}}\right)^2 - 1\right]$$

Some tricks

2)Take $Y_{\rm eq}^2$ out of the bracket

$$\frac{dY}{dx} = -\frac{\langle\sigma v\rangle}{H\,x}\underbrace{sY_{\rm eq}}_{n_{\rm eq}}Y_{\rm eq}\Big[\Big(\frac{Y}{Y_{\rm eq}}\Big)^2 - 1\Big]$$

$$\Rightarrow \frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{n_{\rm eq}\langle\sigma v\rangle}{H}\Big[\Big(\frac{Y}{Y_{\rm eq}}\Big)^2 - 1\Big]$$

Density of targets $\times$ cross section $\times$ velocity = rate of annihilation, $\Gamma_{\rm ann}$

Some tricks

2)Take Y_{eq}^2 out of the bracket

$$\frac{dY}{dx} = -\frac{\langle\sigma v\rangle}{H\,x}\underbrace{sY_{\text{eq}}}_{n_{\text{eq}}}Y_{\text{eq}}\left[\left(\frac{Y}{Y_{\text{eq}}}\right)^2 - 1\right]$$

$$\Rightarrow \frac{x}{Y_{\text{eq}}}\frac{dY}{dx} = -\frac{n_{\text{eq}}\langle\sigma v\rangle}{H}\left[\left(\frac{Y}{Y_{\text{eq}}}\right)^2 - 1\right]$$

Density of targets × cross section × velocity = rate of annihilation, Γ_{ann}

$$\Rightarrow \boxed{\frac{x}{Y_{\text{eq}}}\frac{dY}{dx} = -\frac{\Gamma_{\text{ann}}(x)}{H(x)}\left[\left(\frac{Y}{Y_{\text{eq}}}\right)^2 - 1\right]}$$

Some tricks

2)Take $Y_{\rm eq}^2$ out of the bracket

$$\frac{dY}{dx} = -\frac{\langle\sigma v\rangle}{H\,x}\underbrace{sY_{\rm eq}}_{n_{\rm eq}}Y_{\rm eq}\Big[\Big(\frac{Y}{Y_{\rm eq}}\Big)^2 - 1\Big]$$

$$\Rightarrow \frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{n_{\rm eq}\langle\sigma v\rangle}{H}\Big[\Big(\frac{Y}{Y_{\rm eq}}\Big)^2 - 1\Big]$$

Density of targets × cross section × velocity = rate of annihilation, $\Gamma_{\rm ann}$

$$\Rightarrow \boxed{\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann}(x)}{H(x)}\Big[\Big(\frac{Y}{Y_{\rm eq}}\Big)^2 - 1\Big]}$$

The temperature evolution of the yield is controlled by $\Gamma_{\rm ann}/H$

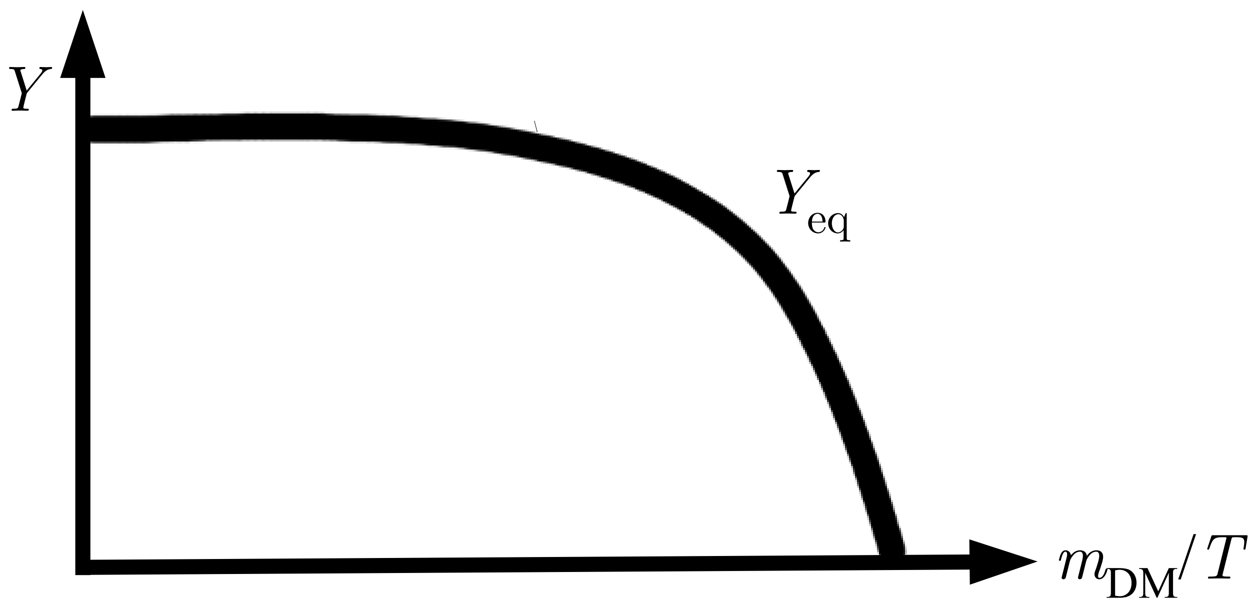
Qualitative behavior of the solution

Remember: $\Gamma_{\rm ann} = n_{\rm eq}\langle\sigma v\rangle$

$$n_{\rm eq} = \begin{cases} \sim T^3 & \text{if } T \gg m_{\rm DM} \\ \sim (m_{\rm DM}T)^{3/2} e^{-m_{\rm DM}/T} & \text{if } T \ll m_{\rm DM} \end{cases}$$

$$H \sim T^2$$

Equilibrium yield:



Qualitative behavior of the solution

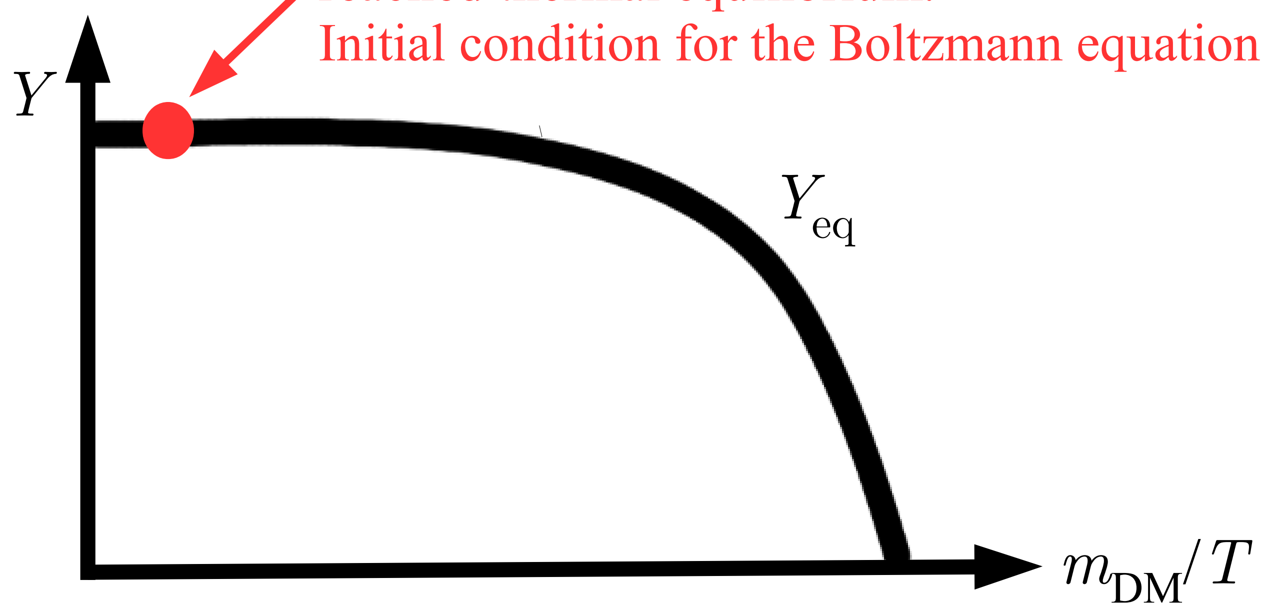
Remember: $\Gamma_{\text{ann}} = n_{\text{eq}} \langle \sigma v \rangle$

$$n_{\text{eq}} = \begin{cases} \sim T^3 & \text{if } T \gg m_{\text{DM}} \\ \sim (m_{\text{DM}} T)^{3/2} e^{-m_{\text{DM}}/T} & \text{if } T \ll m_{\text{DM}} \end{cases}$$

$$H \sim T^2$$

Equilibrium yield:

Assume that at early times the DM reached thermal equilibrium.
Initial condition for the Boltzmann equation

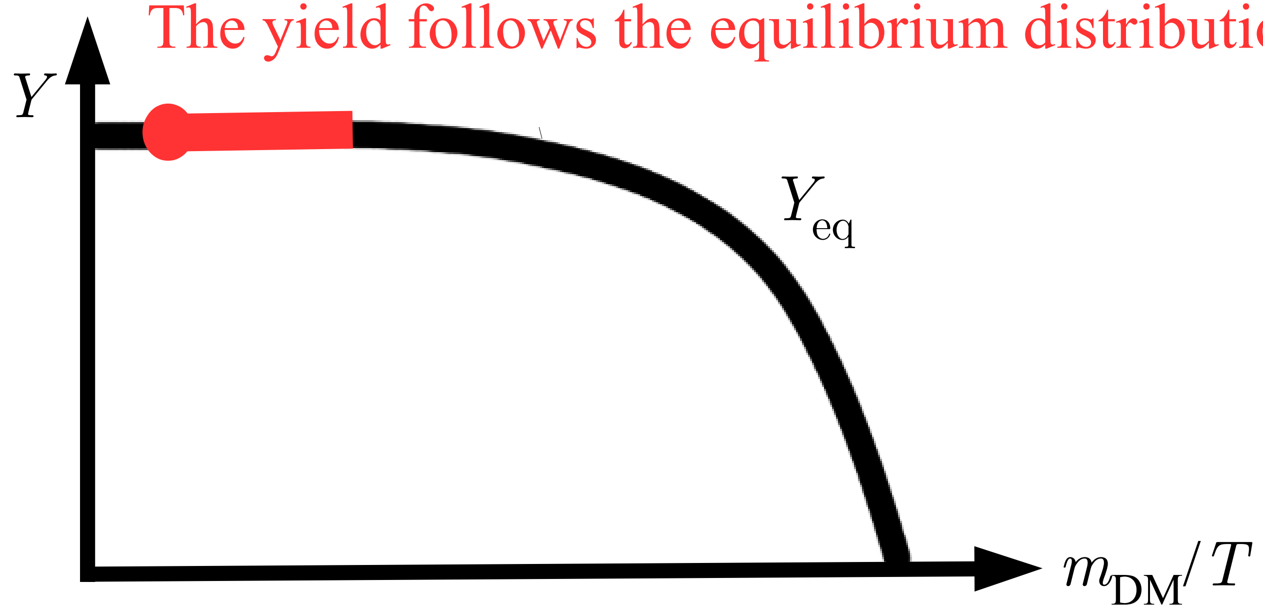


Qualitative behavior of the solution

1) Solution at very early times ($x \ll 1$, or $T \gg m_{\rm DM}$)

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\underbrace{\frac{\Gamma_{\rm ann}(x)}{H(x)}}_{\gg 1}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right]$$

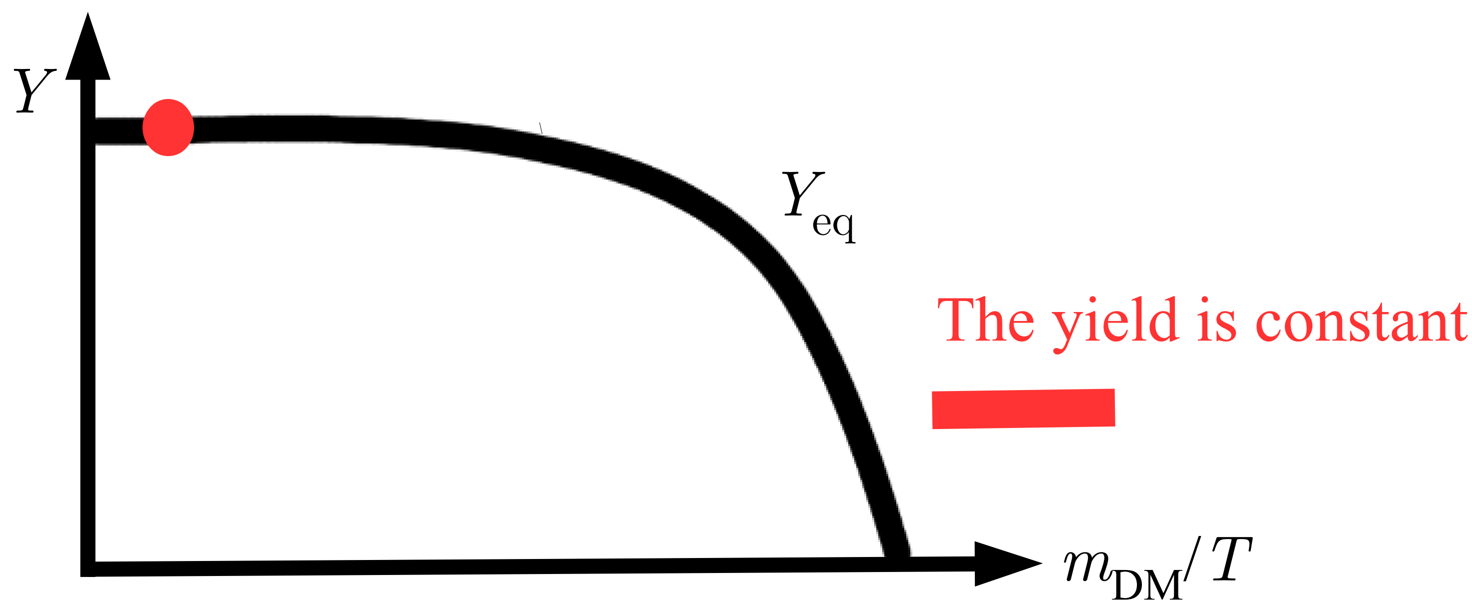
The yield follows the equilibrium distribution



Qualitative behavior of the solution

1) Solution at very late times ($x \gg 1$, or $T \ll m_{\rm DM}$)

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\underbrace{\frac{\Gamma_{\rm ann}(x)}{H(x)}}_{\ll 1}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right] \simeq 0$$

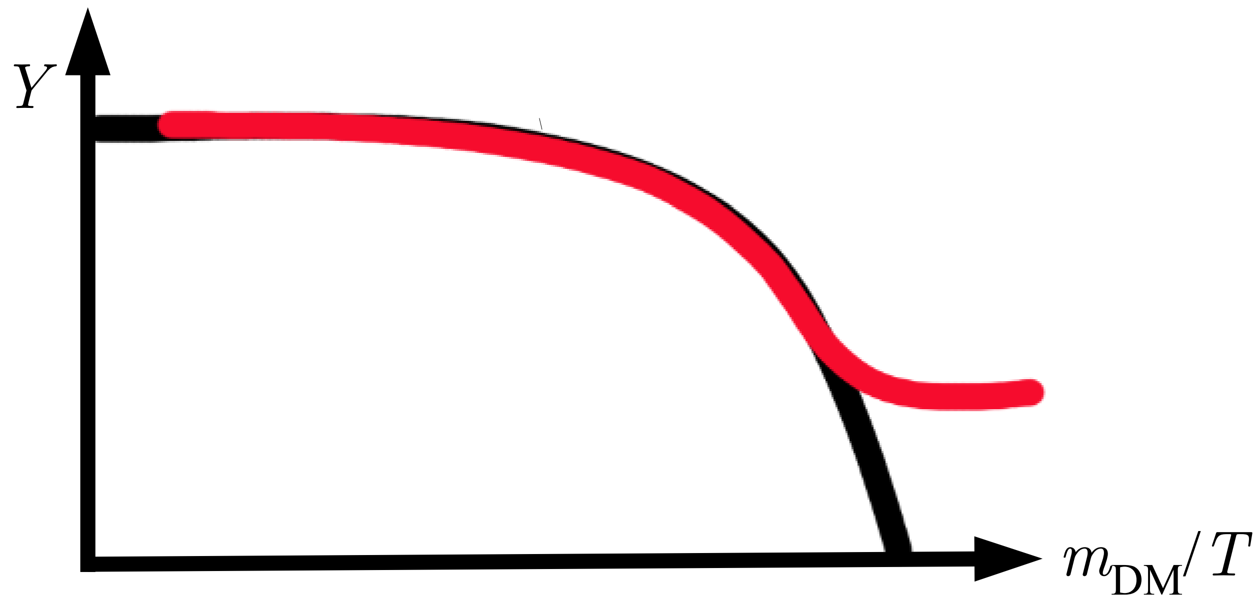


Qualitative behavior of the solution

1) Solution at very late times ($x \gg 1$, or $T \ll m_{\rm DM}$)

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann}(x)}{H(x)}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right] \simeq 0$$

Extrapolation between both solutions

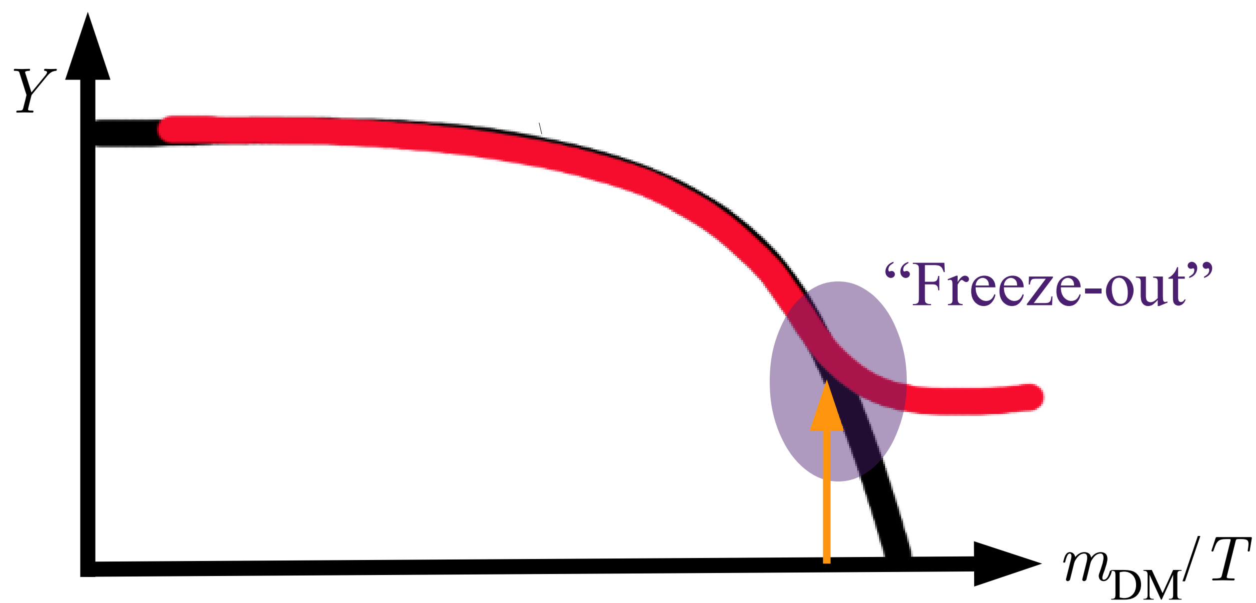


Qualitative behavior of the solution

1) Solution at very late times ($x \gg 1$, or $T \ll m_{\rm DM}$)

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_{\rm ann}(x)}{H(x)}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right] \simeq 0$$

Extrapolation between both solutions



$\dfrac{\Gamma_{\rm ann}}{H} \gg 1$ $\dfrac{\Gamma_{\rm ann}}{H} \simeq 1$ $\dfrac{\Gamma_{\rm ann}}{H} \ll 1$

Freeze-out temperature

Defined from the condition: $\boxed{\frac{\Gamma_{\rm ann}(T_{\rm fo})}{H(T_{\rm fo})} = 1}$

Then,

$$n_{\rm eq}(T_{\rm fo})\langle\sigma v\rangle\Big|_{\rm fo} = H(T_{\rm fo})$$

$$g\left(\frac{m_{\rm DM}T_{\rm fo}}{2\pi}\right)^{3/2} e^{-m_{\rm DM}/T_{\rm fo}}\langle\sigma v\rangle\Big|_{\rm fo} = 1.66\sqrt{g_{\rm eff}}\frac{T_{\rm fo}^2}{M_{\rm Pl}}$$

Finally, one obtains:

$$\boxed{\frac{m_{\rm DM}}{T_{\rm fo}} = \log\left[\frac{g}{1.66\sqrt{g_{\rm eff}}(2\pi)^{3/2}} m_{\rm DM} M_{\rm Pl}\langle\sigma v\rangle\Big|_{\rm fo}\left(\frac{m_{\rm DM}}{T_{\rm fo}}\right)^{1/2}\right]}$$

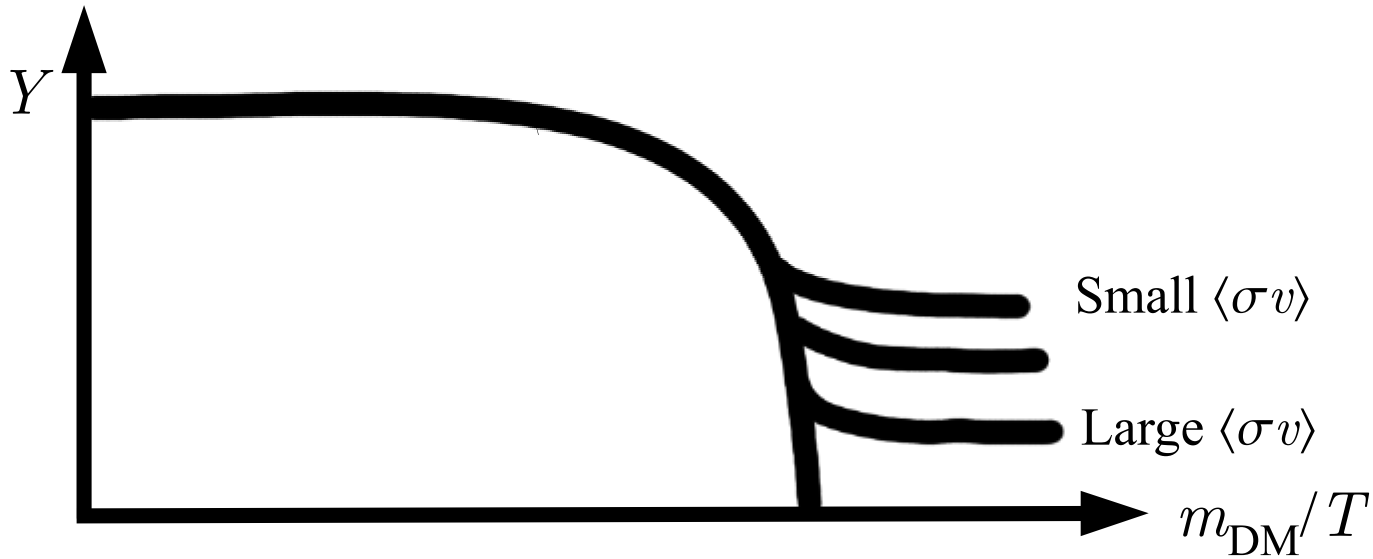
Implicit equation, with a mild dependence on $m_{\rm DM}$. The solution is

$$\boxed{\frac{m_{\rm DM}}{T_{\rm fo}} = 20 - 30}$$

Number density of WIMPs at freeze-out

$$\boxed{n_{\rm eq}(T_{\rm fo}) = \frac{H(T_{\rm fo})}{\langle \sigma v \rangle \big|_{\rm fo}}}$$

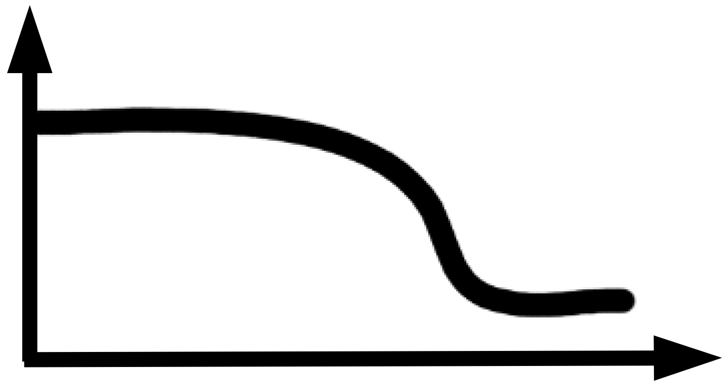
IMPORTANT: the number density of WIMPs at freeze-out is inversely proportional to the annihilation cross section



DM density today

$$\Omega_{\rm DM} = \frac{\rho_{\rm DM}}{\rho_c} = \frac{m_{\rm DM} n_0}{\rho_c} = \frac{m_{\rm DM} s_0 Y_0}{\rho_c}$$

Crucial point: the yield today is the same as the yield at freeze-out



DM density today

$$\Omega_{\rm DM} = \frac{\rho_{\rm DM}}{\rho_c} = \frac{m_{\rm DM} n_0}{\rho_c} = \frac{m_{\rm DM} s_0 Y_0}{\rho_c}$$

$$= \frac{m_{\rm DM} s_0 Y_{\rm fo}}{\rho_c}$$

Relic density

$$\Omega_{\rm DM} = \frac{\rho_{\rm DM}}{\rho_c} = \frac{m_{\rm DM} n_0}{\rho_c} = \frac{m_{\rm DM} s_0 Y_0}{\rho_c}$$

$$= \frac{m_{\rm DM} s_0 Y_{\rm fo}}{\rho_c} = \frac{m_{\rm DM} s_0}{\rho_c} \frac{n_{\rm eq}(T_{fo})}{s(T_{\rm fo})}$$

$$= \frac{m_{\rm DM} s_0}{\rho_c} \frac{H(T_{\rm fo})}{s(T_{\rm fo})\langle\sigma v\rangle\big|_{\rm fo}}$$

Use:

$$s = h_{\rm eff}(T)\frac{2\pi^2}{45}T^3$$

$$s_0 \simeq 3000\,{\rm cm}^{-3}$$

$$H = 1.66\sqrt{g_{\rm eff}}\frac{T^2}{M_{\rm Pl}}$$

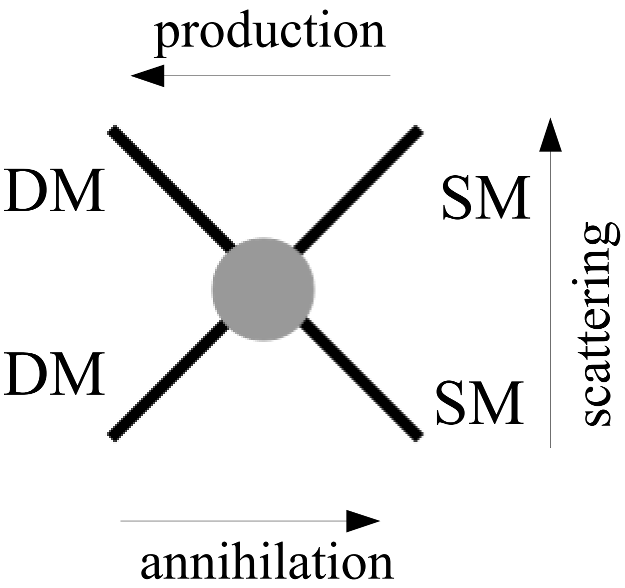
$$\rho_c = \frac{3H_0^2}{8\pi G} = 1.05h^2 \times 10^{-5}\,{\rm GeV\,cm}^{-3}$$

$$T_{\rm fo} \simeq \frac{m_{\rm DM}}{25}$$

$$\boxed{\Omega_{\rm DM}h^2 \simeq \frac{3\times 10^{-27}\,{\rm cm}^3\,{\rm s}^{-1}}{\langle\sigma v\rangle\big|_{\rm fo}}}$$

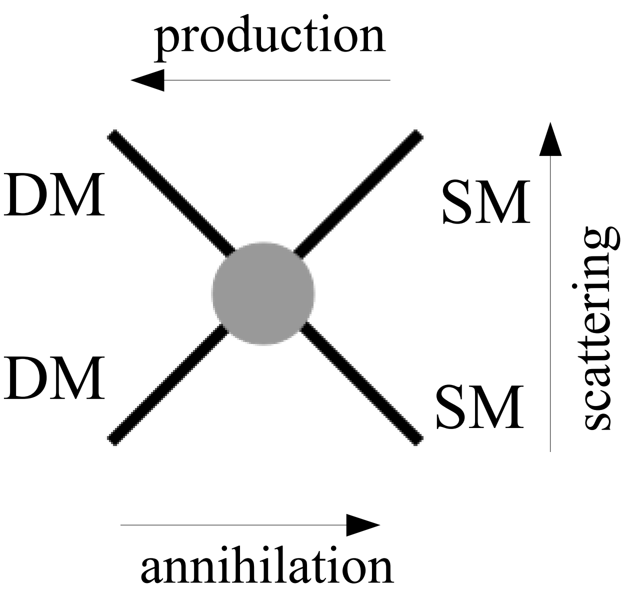
Main results from this part

WIMP dark matter



Main results from this part

WIMP dark matter

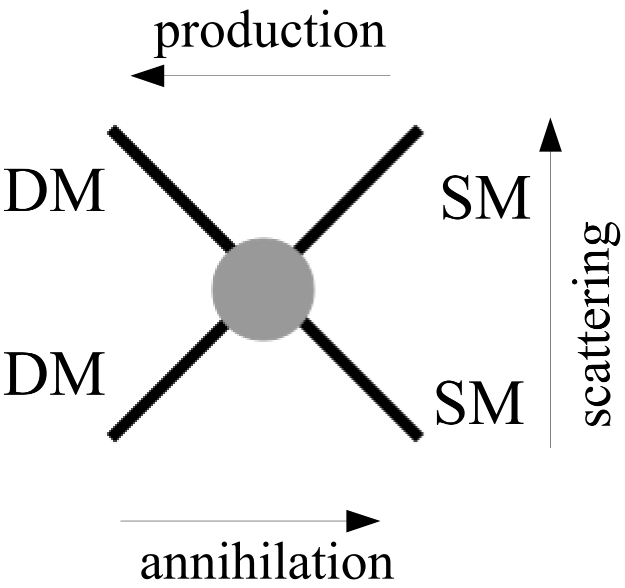


Relic abundance of DM particles

$$\boxed{\Omega h^2 \simeq \frac{3\times 10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle\sigma v\rangle}}$$

Main results from this part

WIMP dark matter



Relic abundance of DM particles

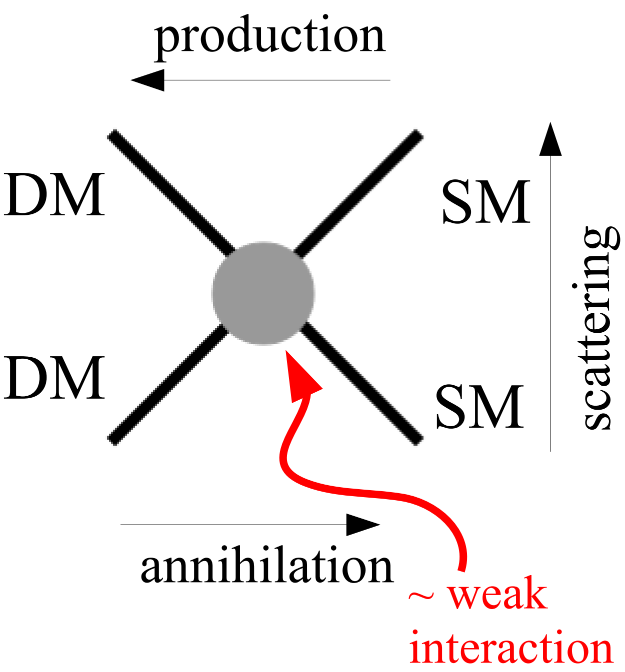
$$\boxed{\Omega h^2 \simeq \frac{3\times 10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle\sigma v\rangle}}$$

Correct DM abundance Ωh^2=0.12 if

$$\boxed{\langle\sigma v\rangle \simeq 3\times 10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1} = 1\,\mathrm{pb}\cdot c}$$

Main results from this part

WIMP dark matter



Relic abundance of DM particles

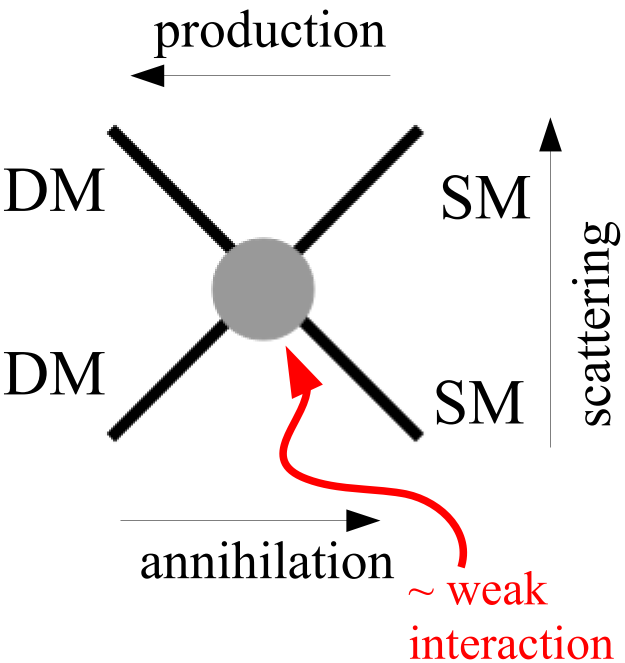
$$\boxed{\Omega h^2 \simeq \frac{3\times 10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle \sigma v\rangle}}$$

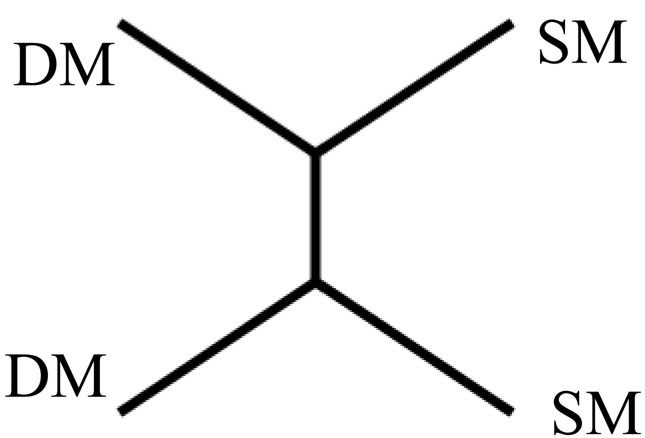
Correct DM abundance Ωh^2=0.12 if

$$\boxed{\langle \sigma v\rangle \simeq 3\times 10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1} = 1\,\mathrm{pb}\cdot c}$$

Main results from this part

WIMP dark matter





Relic abundance of DM particles

$$\boxed{\Omega h^2 \simeq \frac{3 \times 10^{-27}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle \sigma v \rangle}}$$

Correct DM abundance Ωh^2=0.12 if

$$\boxed{\langle \sigma v \rangle \simeq 3 \times 10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1} = 1\,\mathrm{pb} \cdot c}$$

$$\sigma \sim \frac{g^4}{m_{\mathrm{DM}}^2} = 1\,\mathrm{pb}$$

$$\boxed{m_{\mathrm{DM}} \sim 10\,\mathrm{GeV} - 1\,\mathrm{TeV}}$$

(provided $g \sim g_{\mathrm{weak}} \sim 0.1$)

LUX

CMS

ATLAS

MAGIC

Xe
XENON
Dark Matter Project

fermi
Gamma-ray
Space Telescope

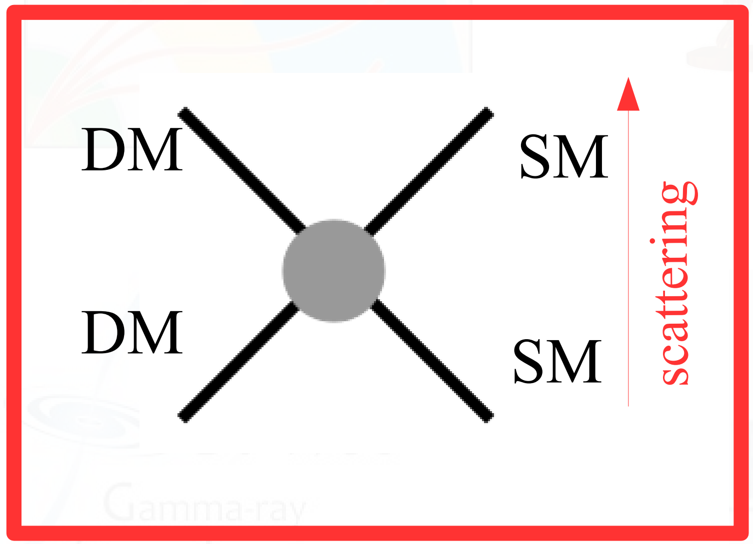
PaMeLa

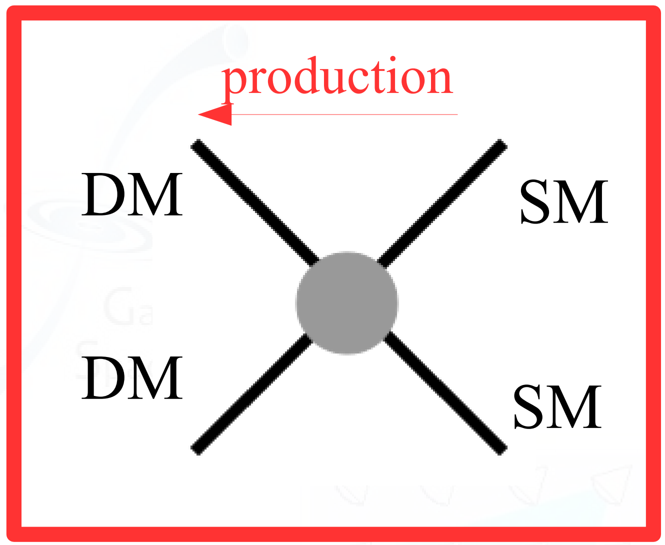
HESS

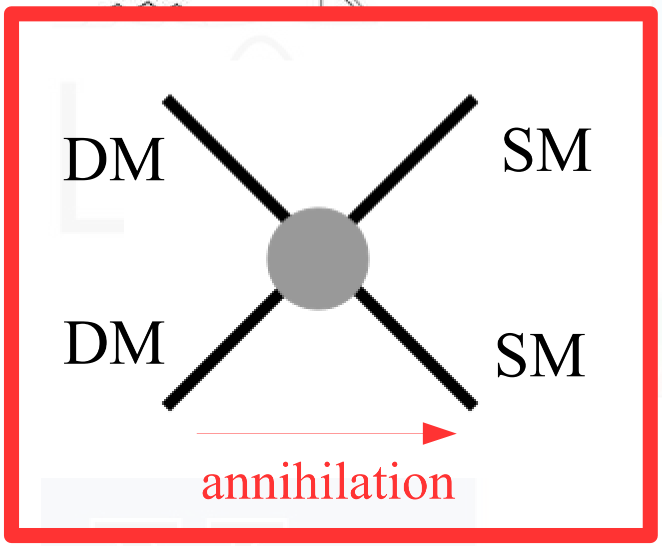
Alpha Magnetic Spectrometer
AMS-02
Europe • Asia
North America

VERITAS

ICECUBE
SOUTH POLE NEUTRINO OBSERVATORY


DM
DM
SM
SM
scattering


production
DM
DM
SM
SM



MAGIC

Fermi
Gamma-ray Space Telescope

PaMeLa

H E S S

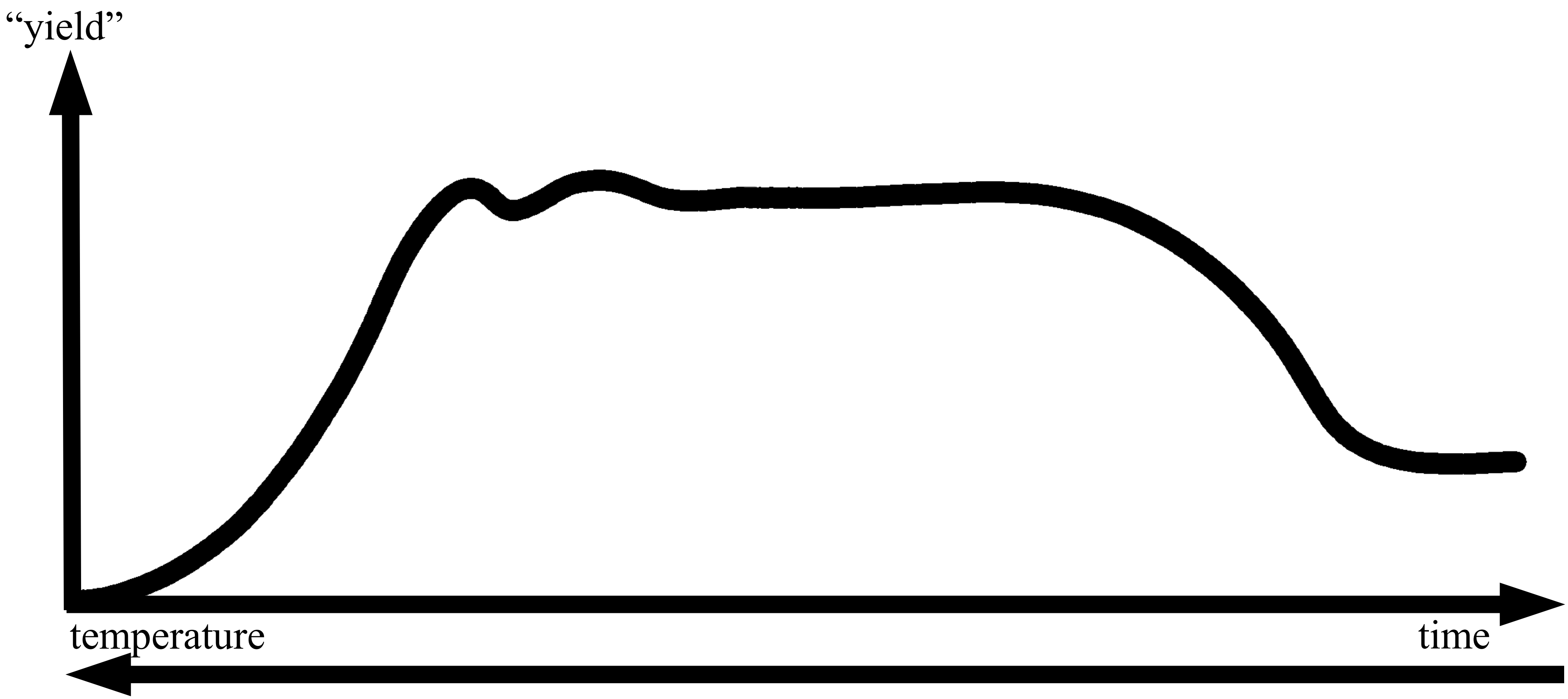
AMS-02

VERITAS

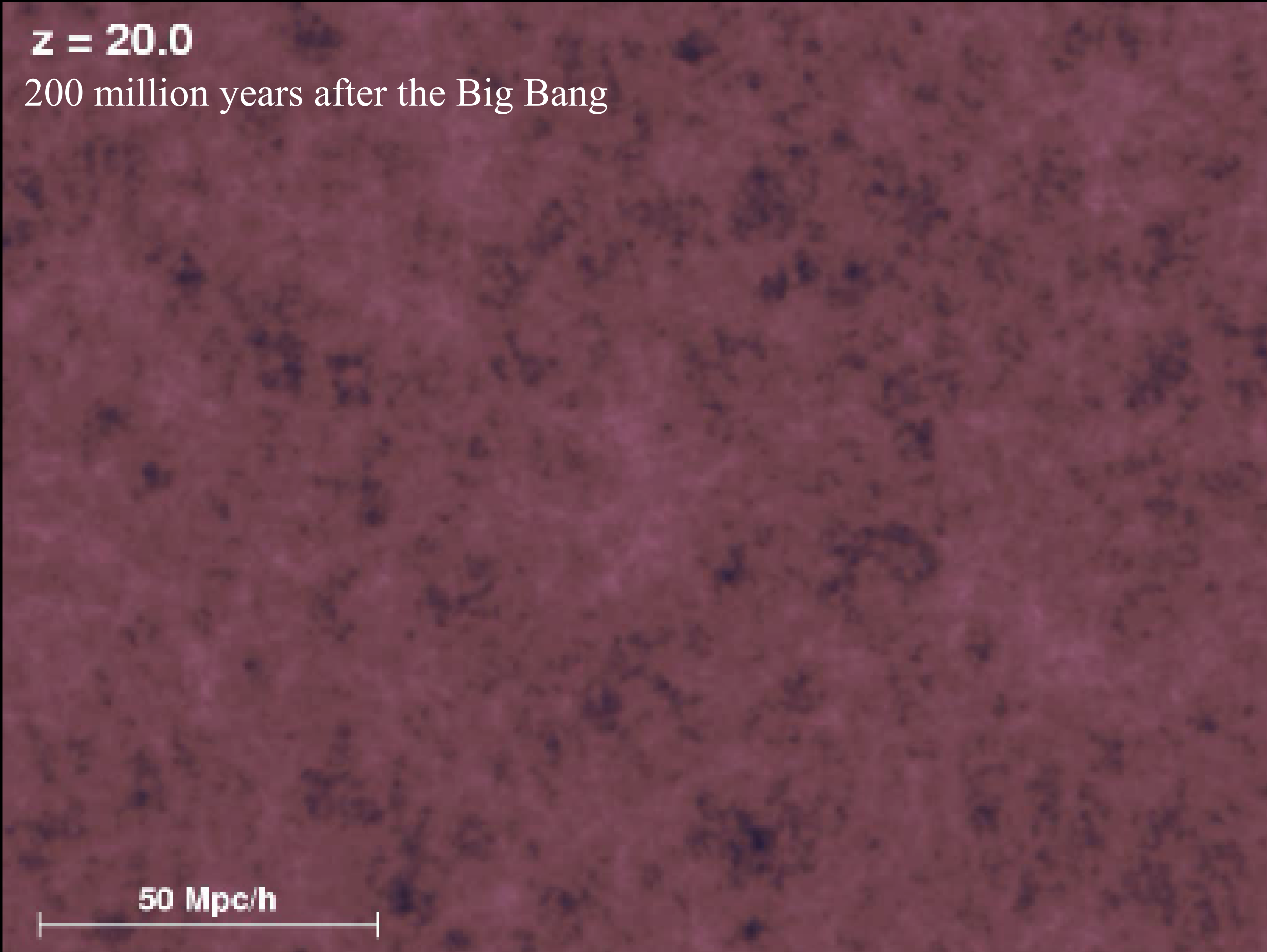
ICECUBE
SOUTH POLE NEUTRINO OBSERVATORY

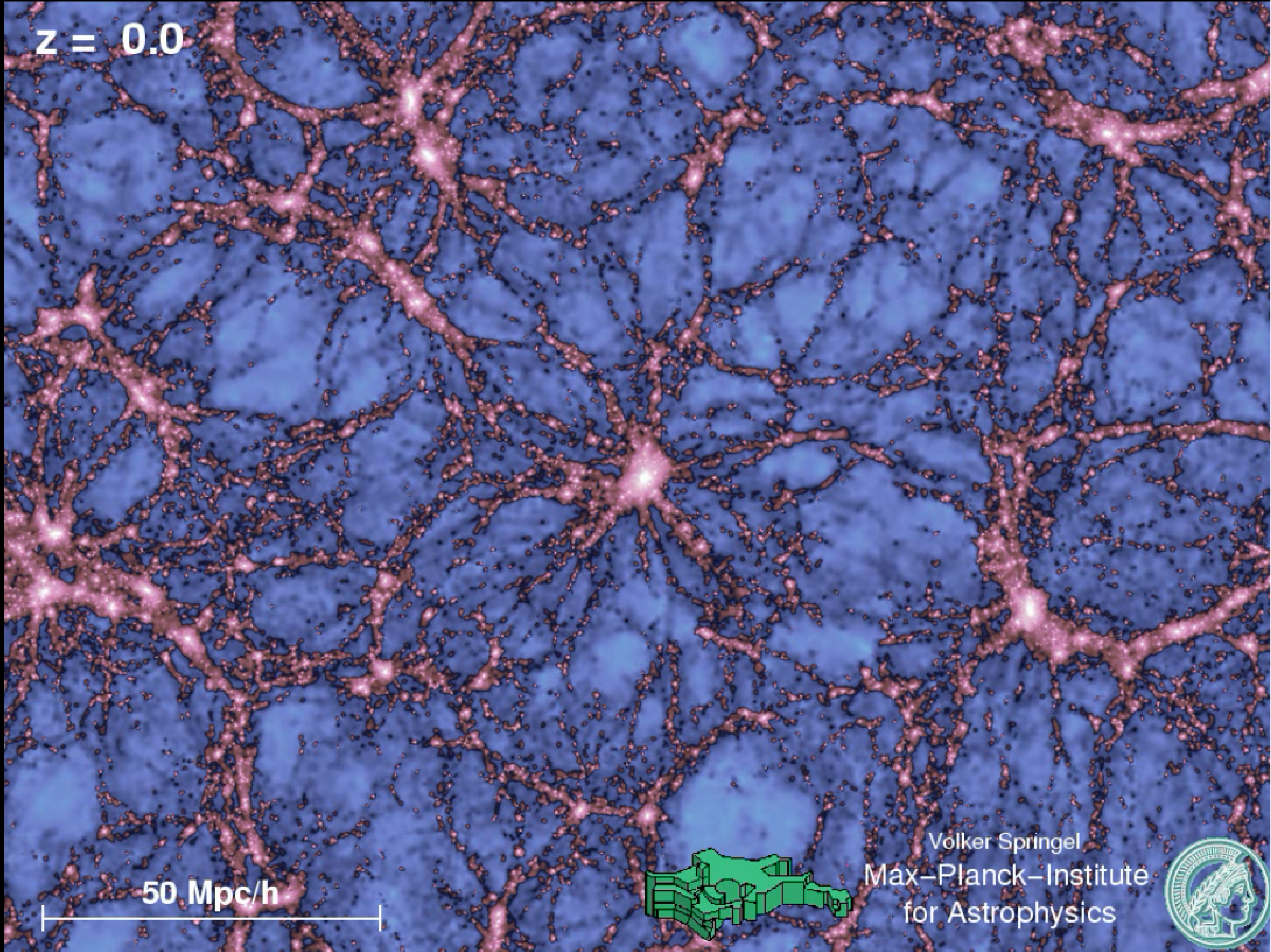
Dark matter distribution

WIMP history (in a nutshell)



The universe at T~1 GeV

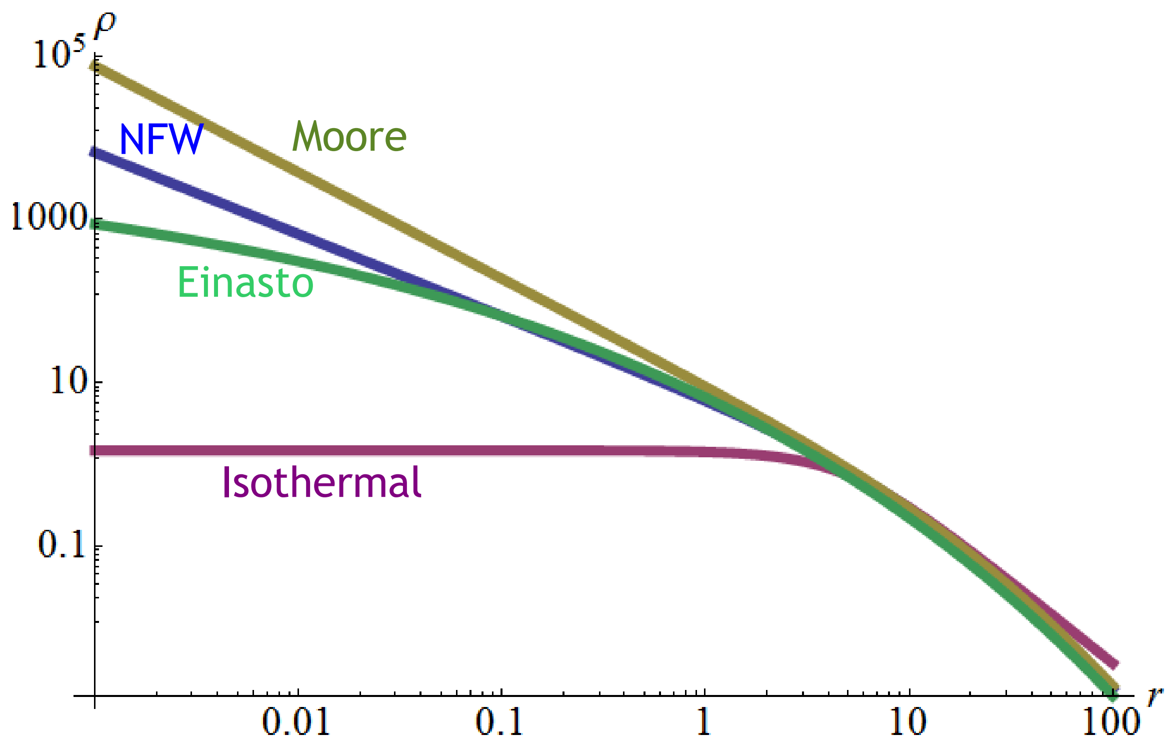

z = 20.0
200 million years after the Big Bang
50 Mpc/h


z = 0.0
50 Mpc/h
Volker Springel
Max-Planck-Institute
for Astrophysics

z=0.0
Distance Sun to Milky Way Center ~ 8.5 kpc
8 kpc

Density distribution of dark matter particles:

- Assume spherical symmetry (in a first approximation).
- Radial distribution:



NFW, Isothermal, Moore

$$\rho(r) = \frac{\rho_0}{(r/r_c)^\gamma [1 + (r/r_c)^\alpha]^{(\beta-\gamma)/\alpha}}$$

Halo model	α	β	γ	r_c (kpc)
Navarro, Frenk, White	1	3	1	20
Isothermal	2	2	0	3.5
Moore	1.5	3	1.5	28

Einasto

$$\rho(r) = \rho_0 \exp\left[-\frac{2}{\alpha}\left(\left(\frac{r}{r_s}\right)^\alpha - 1\right)\right]$$

$\alpha = 0.17, \ r_s = 20$ kpc

- Normalized such that the local DM density is

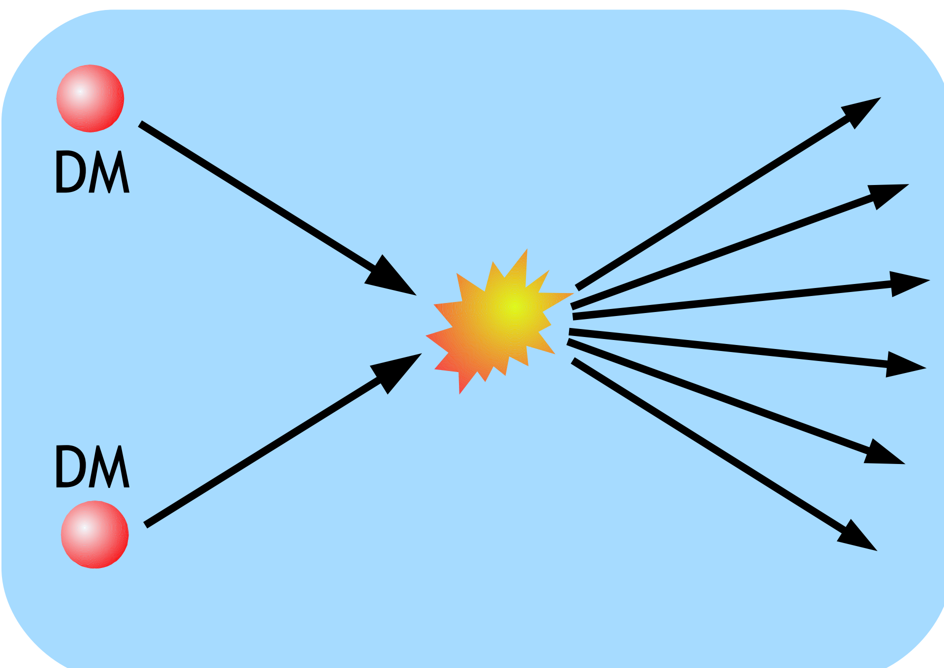
$\rho(r=8.5\text{ kpc}) = 0.38\text{ GeV/cm}^3$

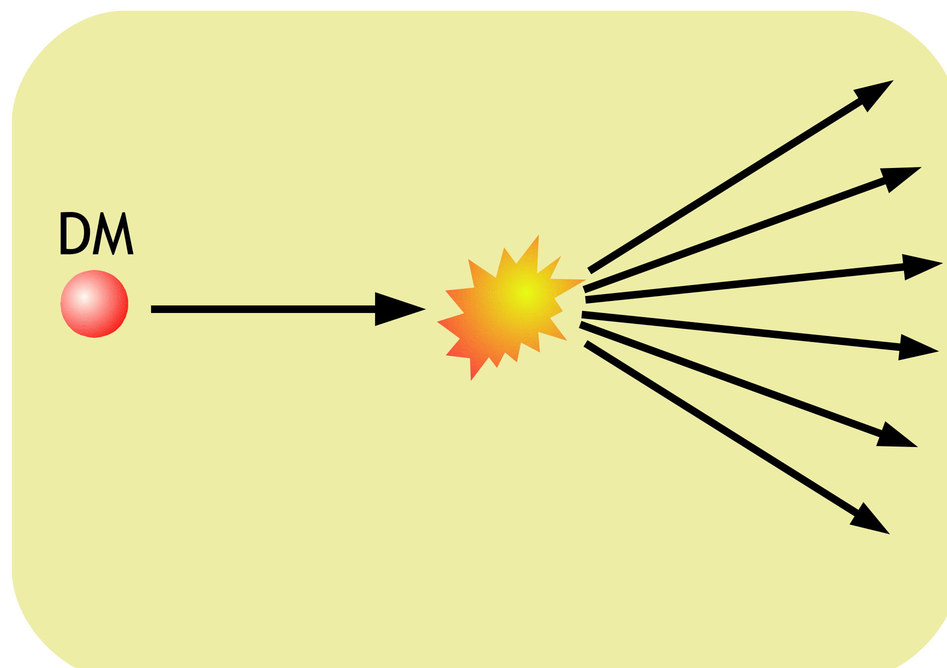
Indirect Dark Matter Searches

Indirect dark matter searches

General idea:

1) Dark matter particles annihilate or decay producing a flux of stable particles: photons, electrons, protons, positrons, antiprotons or (anti-)neutrinos.





Indirect dark matter searches

General idea:

1) Dark matter particles annihilate or decay producing a flux of stable particles: photons, electrons, protons, positrons, antiprotons or (anti-)neutrinos.

2) These particles propagate through the galaxy and through the Solar System. Some of them will reach the Earth.

Indirect dark matter searches

General idea:

1) Dark matter particles annihilate or decay producing a flux of stable particles: photons, electrons, protons, positrons, antiprotons or (anti-)neutrinos.

2) These particles propagate through the galaxy and through the Solar System. Some of them will reach the Earth.

3) The products of the dark matter annihilations or decays are detected together with other particles produced in astrophysical processes (for example, cosmic ray collisions with nuclei in the interstellar medium). The existence of dark matter can then be inferred if there is a significant excess in the fluxes compared to the expected astrophysical backgrounds.

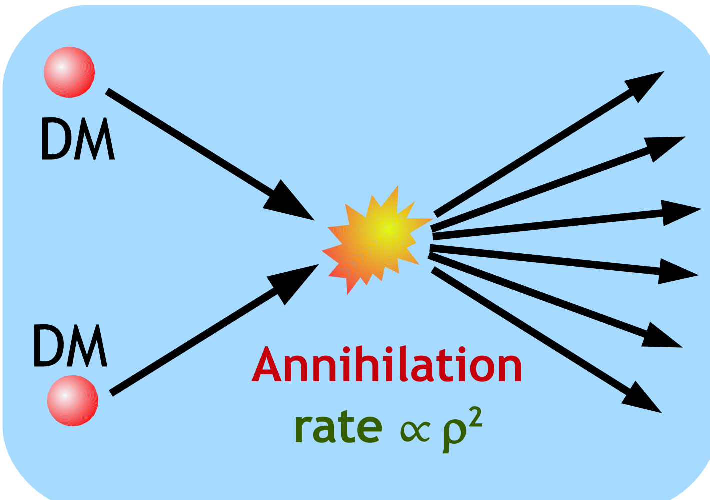
Indirect dark matter searches

Production			Antimatter
Propagation	}	of	{ Gamma-rays
Detection			Neutrinos

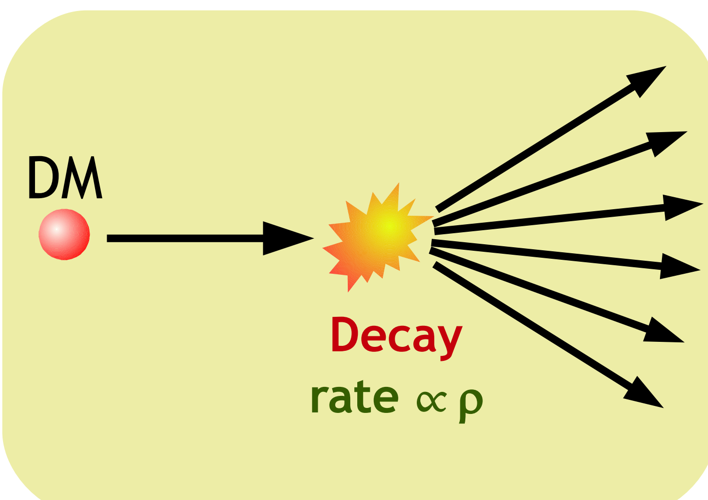
Antimatter

Production

The production is described by the source function: number of particles produced at a given position per unit volume, unit time and unit energy.



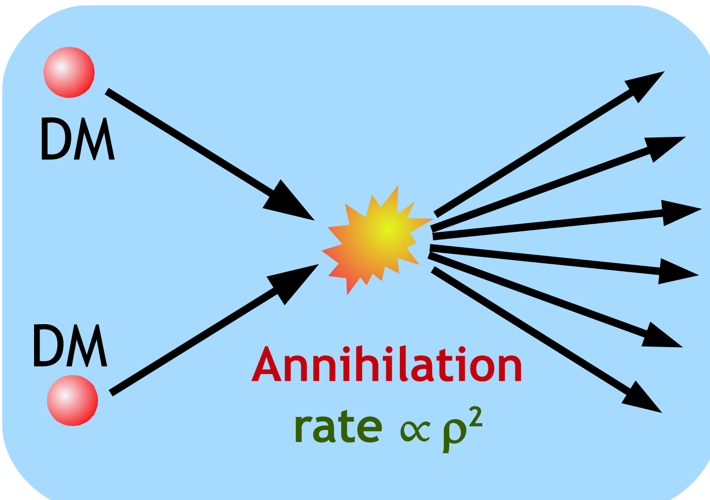
$$Q(E,\vec{r}) = \frac{1}{2}\frac{\rho^2(\vec{r})}{m_{\rm DM}^2}\langle\sigma v\rangle\frac{dN}{dE}$$



$$Q(E,\vec{r}) = \frac{\rho(\vec{r})}{m_{\rm DM}}\frac{1}{\tau_{\rm DM}}\frac{dN}{dE}$$

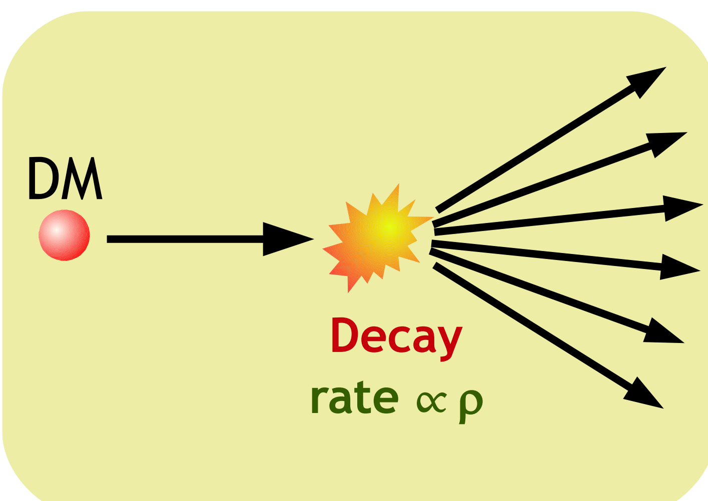
Production

The production is described by the source function: number of particles produced at a given position per unit volume, unit time and unit energy.



$$Q(E,\vec{r}) = \frac{1}{2}\frac{\rho^2(\vec{r})}{m_{\rm DM}^2}\langle\sigma v\rangle\frac{dN}{dE}$$

???

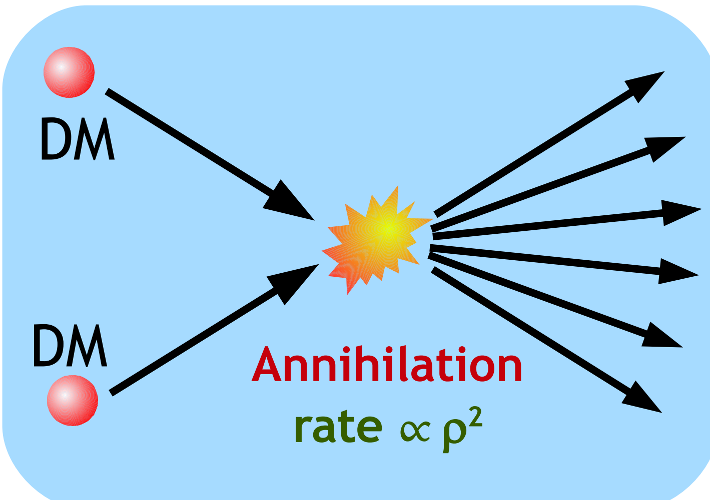


$$Q(E,\vec{r}) = \frac{\rho(\vec{r})}{m_{\rm DM}}\frac{1}{\tau_{\rm DM}}\frac{dN}{dE}$$

???

Production

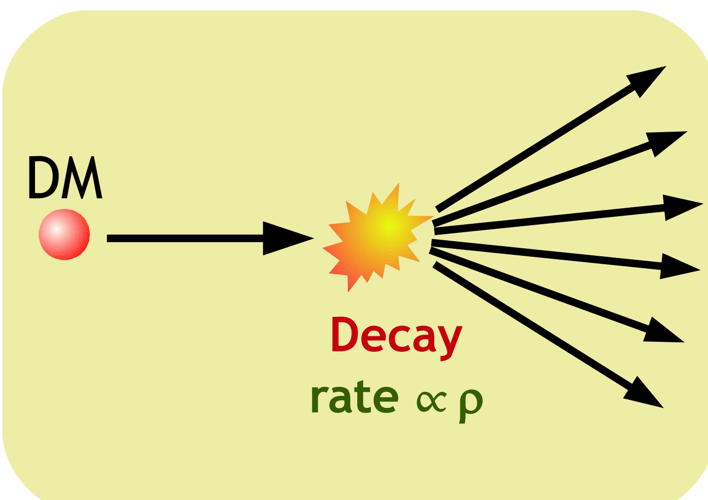
The production is described by the source function: number of particles produced at a given position per unit volume, unit time and unit energy.



$$Q(E,\vec{r}) = \frac{1}{2}\frac{\rho^2(\vec{r})}{m_{\rm DM}^2}\langle\sigma v\rangle\frac{dN}{dE}$$

A well motivated choice:

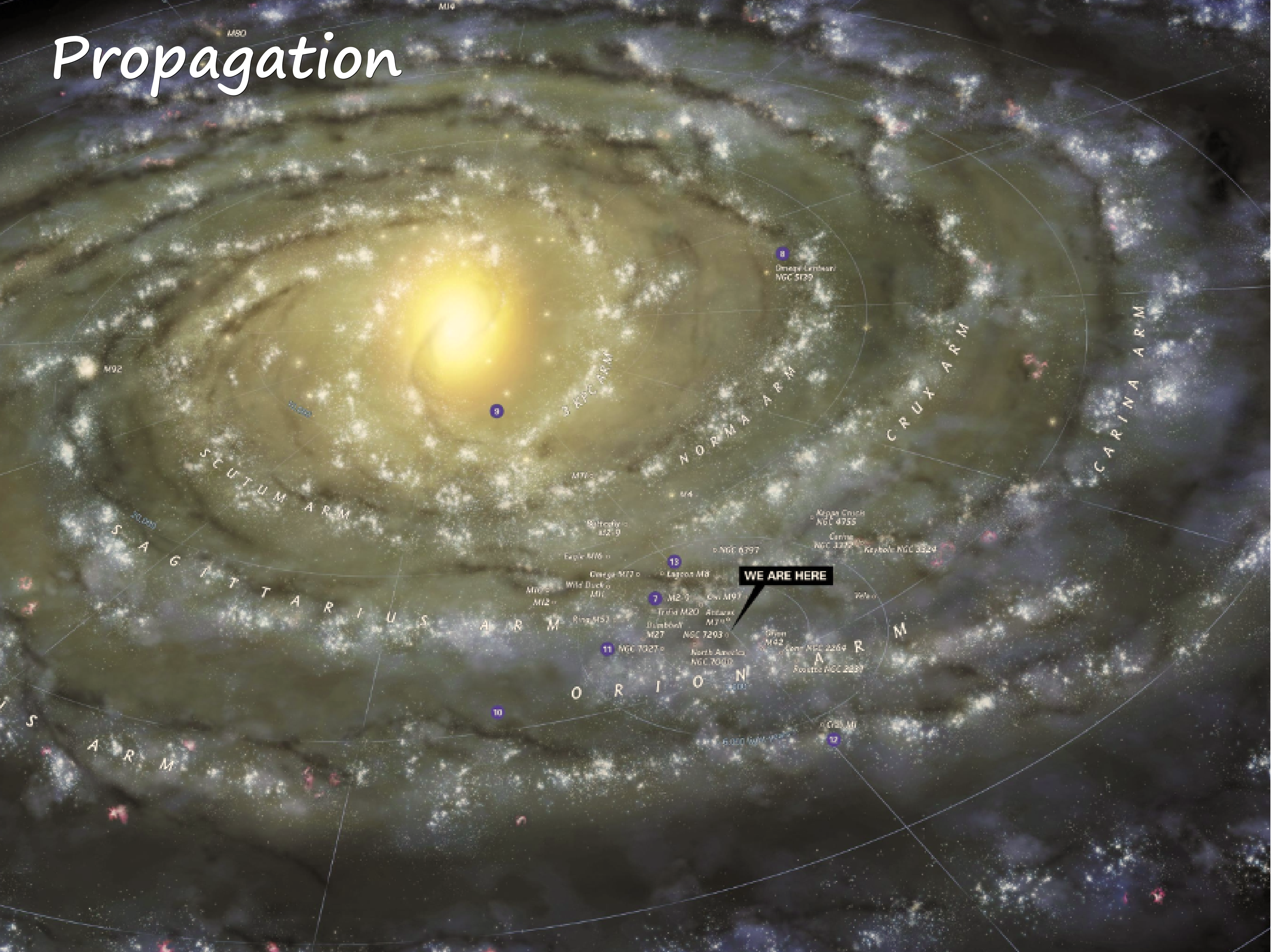
$$(\sigma v) \simeq 3\times10^{-26}\,{\rm cm}^3\,{\rm s}^{-1}$$



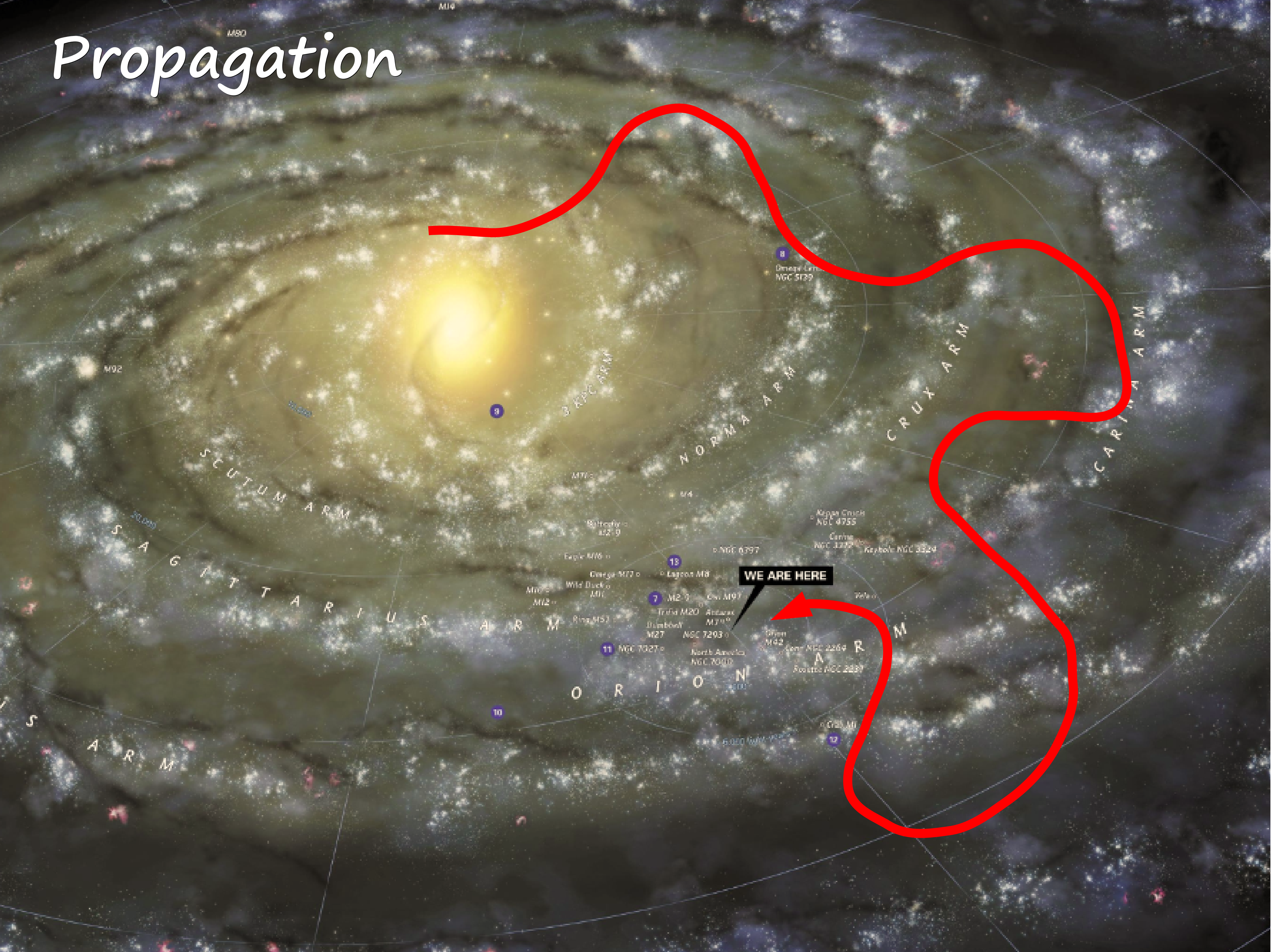
$$Q(E,\vec{r}) = \frac{\rho(\vec{r})}{m_{\rm DM}}\frac{1}{\tau_{\rm DM}}\frac{dN}{dE}$$

???

Propagation

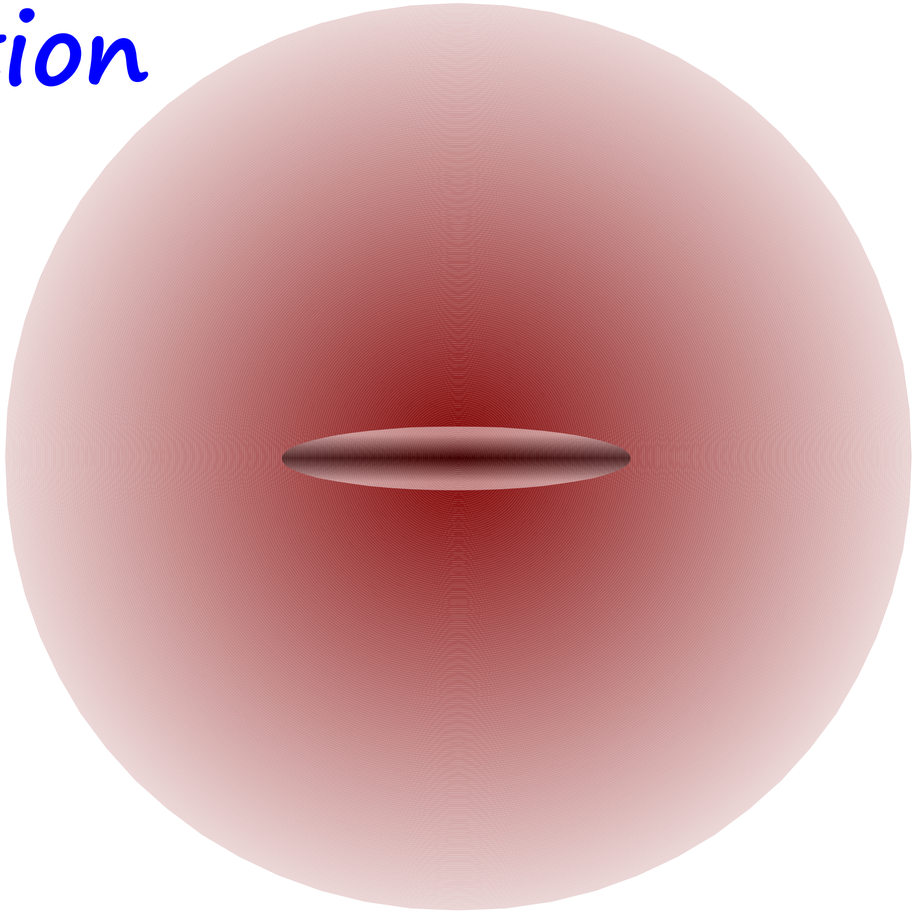


Propagation

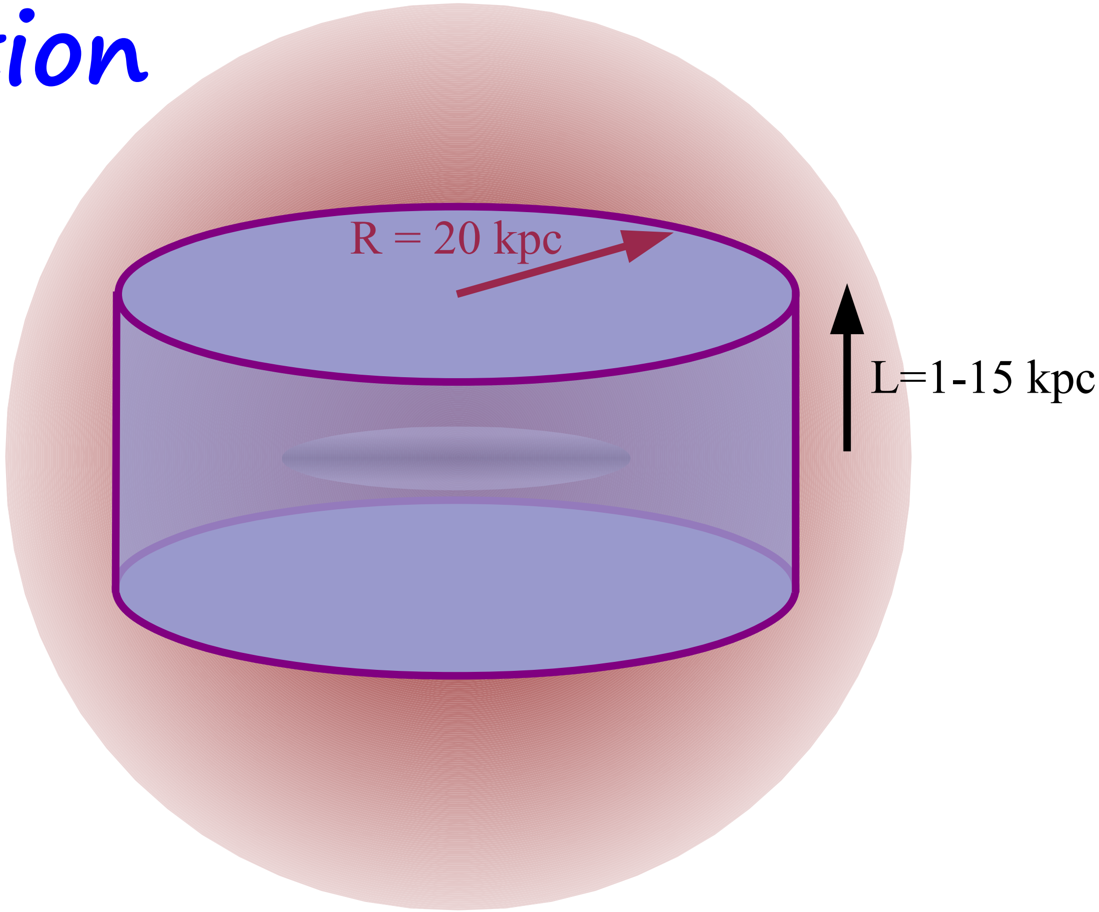


Propagation

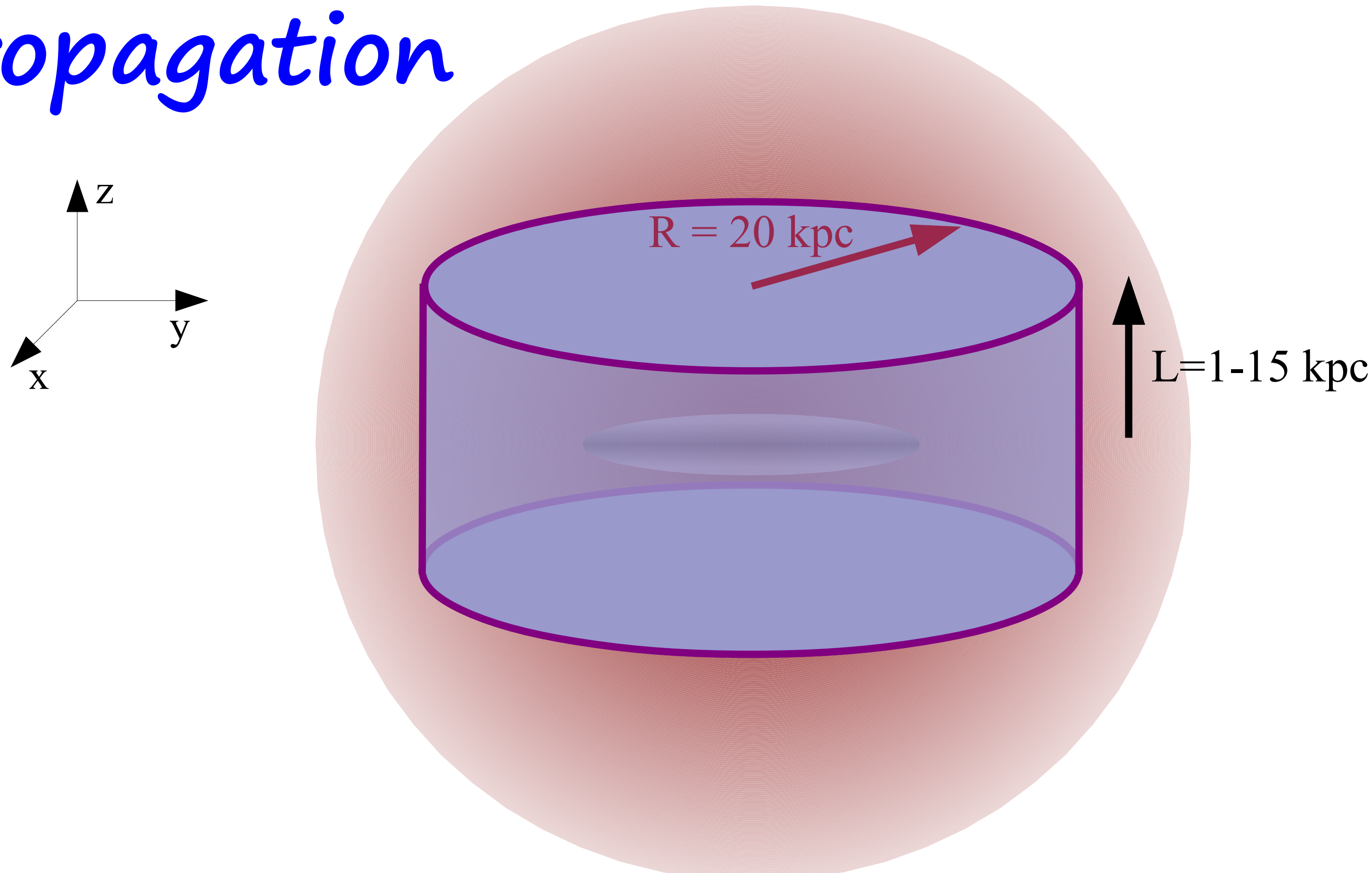
Propagation



Propagation



Propagation



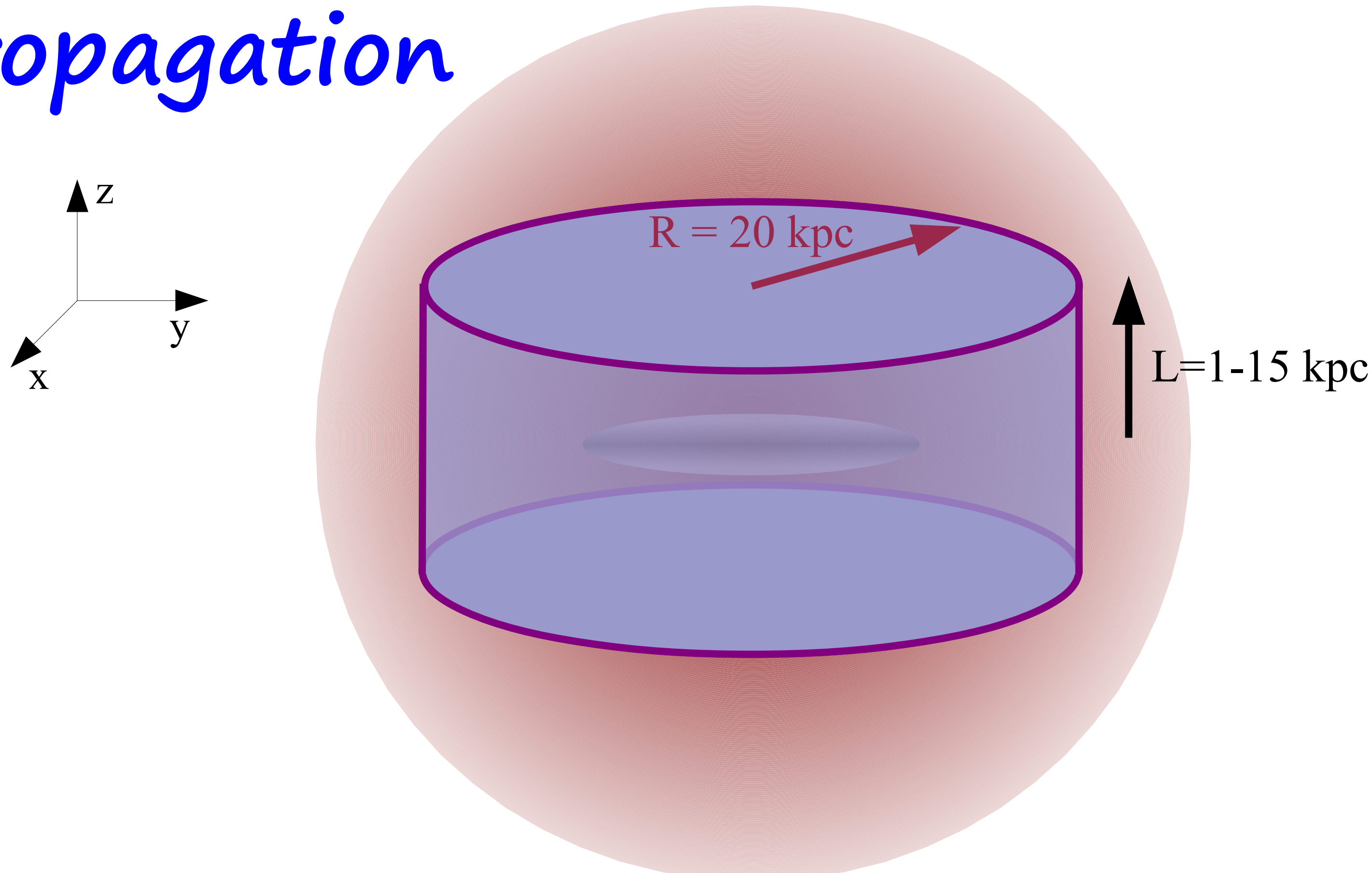
$$0 = \frac{\partial f}{\partial t} = \nabla \cdot [K(T,\vec{r})\nabla f] + \frac{\partial}{\partial T}[b(T,\vec{r})f] - \nabla \cdot [\vec{V}_c(\vec{r})f] - 2h\delta(z)\Gamma_{\rm ann} f + Q(T,\vec{r}) \ .$$

f: number density of antiparticles per unit kinetic energy

interstellar antimatter flux:

$$\Phi^{\rm IS}(T) = \frac{dN}{dt\, dS\, dT\, d\Omega} = \frac{v}{4\pi} f(T)$$

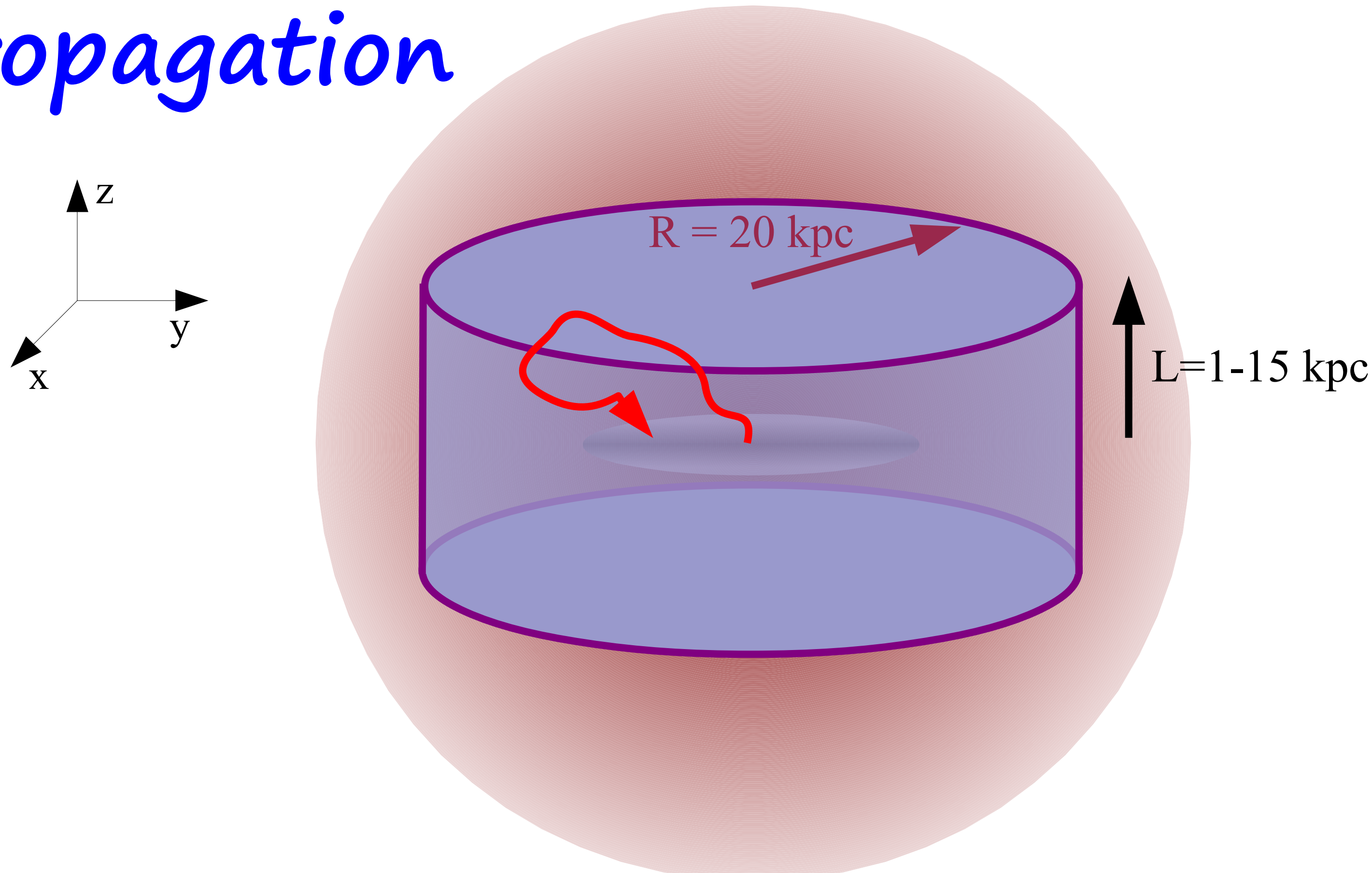
Propagation



$$0 = \frac{\partial f}{\partial t} = \nabla \cdot [K(T,\vec{r})\nabla f] + \frac{\partial}{\partial T}[b(T,\vec{r})f] - \nabla \cdot [\vec{V}_c(\vec{r})f] - 2h\delta(z)\Gamma_{\rm ann} f \boxed{+ Q(T,\vec{r})\ .}$$

Source term $\quad Q(T,\vec{r}) = \begin{cases} \dfrac{1}{2}\dfrac{\rho^2(\vec{r})}{m_{\rm DM}^2}\langle\sigma v\rangle \dfrac{dN}{dT} & \text{dark matter annihilation} \\[2ex] \dfrac{\rho(\vec{r})}{m_{\rm DM}}\dfrac{1}{\tau_{\rm DM}}\dfrac{dN}{dE} & \text{dark matter decay} \end{cases}$

Propagation

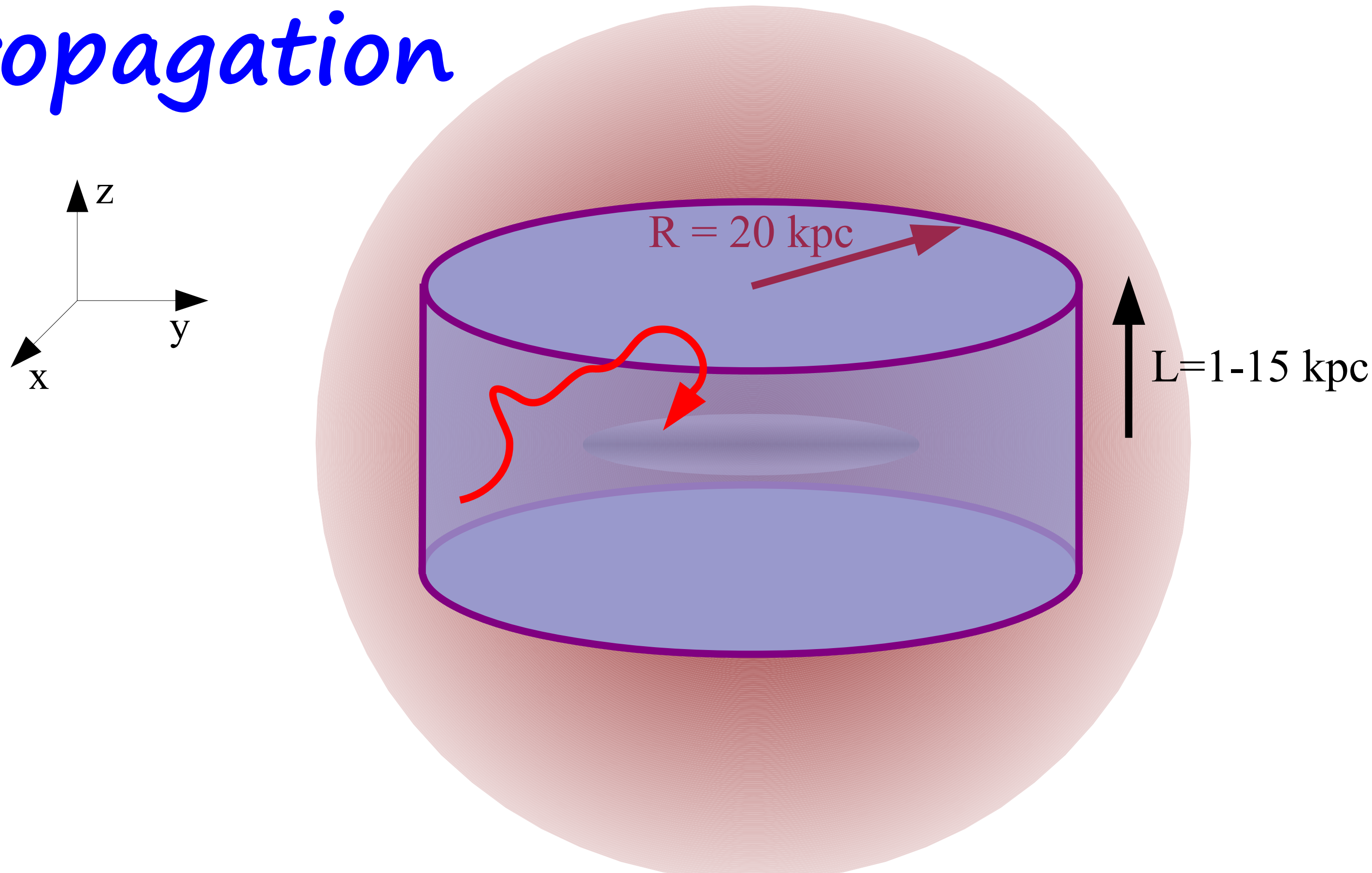


$$0 = \frac{\partial f}{\partial t} = \nabla \cdot [K(T,\vec{r})\nabla f] + \frac{\partial}{\partial T}[b(T,\vec{r})f] - \nabla \cdot [\vec{V}_c(\vec{r})f] - 2h\delta(z)\Gamma_{\rm ann} f \boxed{+ Q(T,\vec{r})\ .}$$

Source term

$$Q(T,\vec{r}) = \begin{cases} \dfrac{1}{2}\dfrac{\rho^2(\vec{r})}{m_{\rm DM}^2}\langle\sigma v\rangle \dfrac{dN}{dT} & \text{dark matter annihilation} \\[2ex] \dfrac{\rho(\vec{r})}{m_{\rm DM}}\dfrac{1}{\tau_{\rm DM}}\dfrac{dN}{dE} & \text{dark matter decay} \end{cases}$$

Propagation

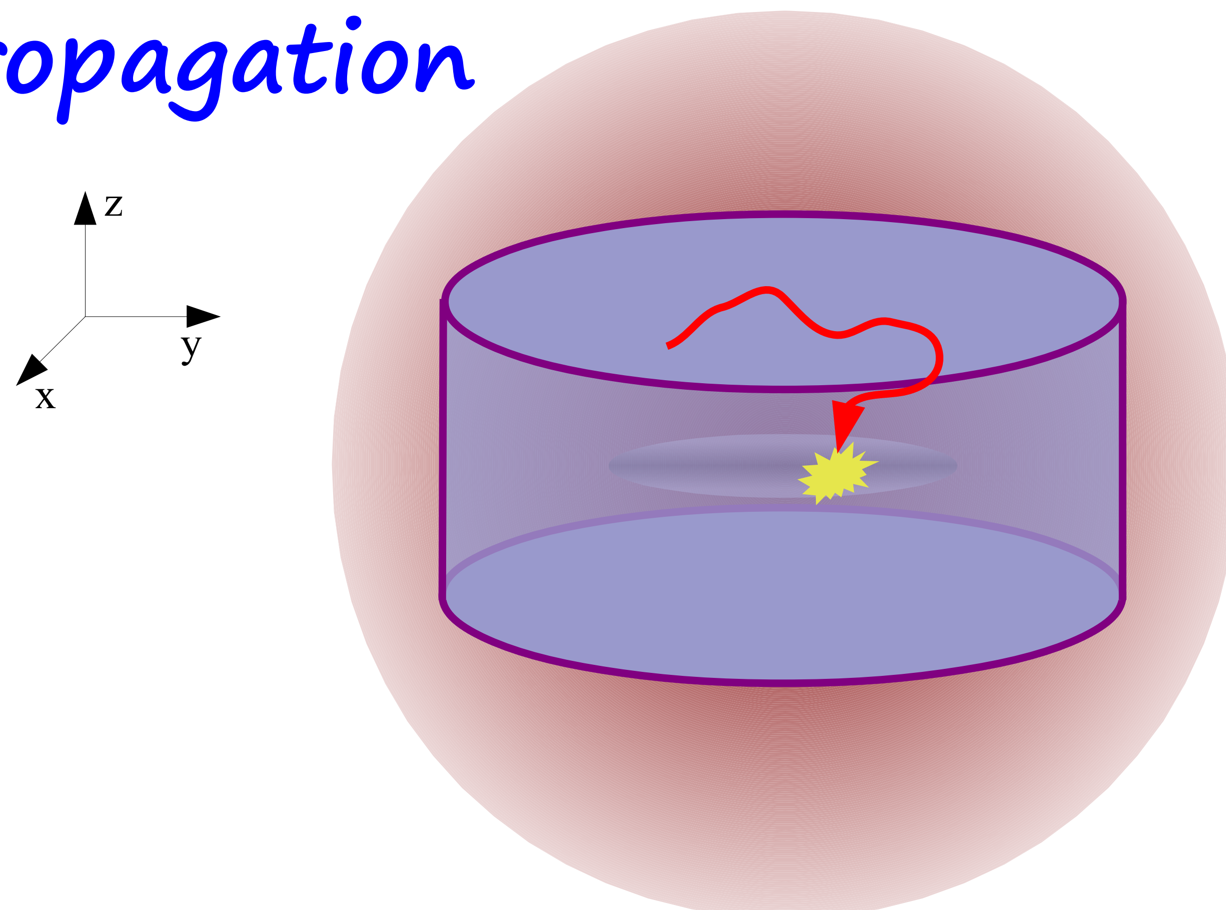


$$0 = \frac{\partial f}{\partial t} = \nabla \cdot [K(T,\vec{r})\nabla f] + \frac{\partial}{\partial T}[b(T,\vec{r})f] - \nabla \cdot [\vec{V}_c(\vec{r})f] - 2h\delta(z)\Gamma_{\rm ann} f \boxed{+ Q(T,\vec{r})\ .}$$

Source term

$$Q(T,\vec{r}) = \begin{cases} \dfrac{1}{2}\dfrac{\rho^2(\vec{r})}{m_{\rm DM}^2}\langle\sigma v\rangle\dfrac{dN}{dT} & \text{dark matter annihilation} \\[2ex] \dfrac{\rho(\vec{r})}{m_{\rm DM}}\dfrac{1}{\tau_{\rm DM}}\dfrac{dN}{dE} & \text{dark matter decay} \end{cases}$$

Propagation



$$0 = \frac{\partial f}{\partial t} = \nabla \cdot [K(T,\vec{r})\nabla f] + \frac{\partial}{\partial T}[b(T,\vec{r})f] - \nabla \cdot [\vec{V}_c(\vec{r})f] \boxed{- 2h\delta(z)\Gamma_{\rm ann} f} + Q(T,\vec{r}) \ .$$

Annihilation term

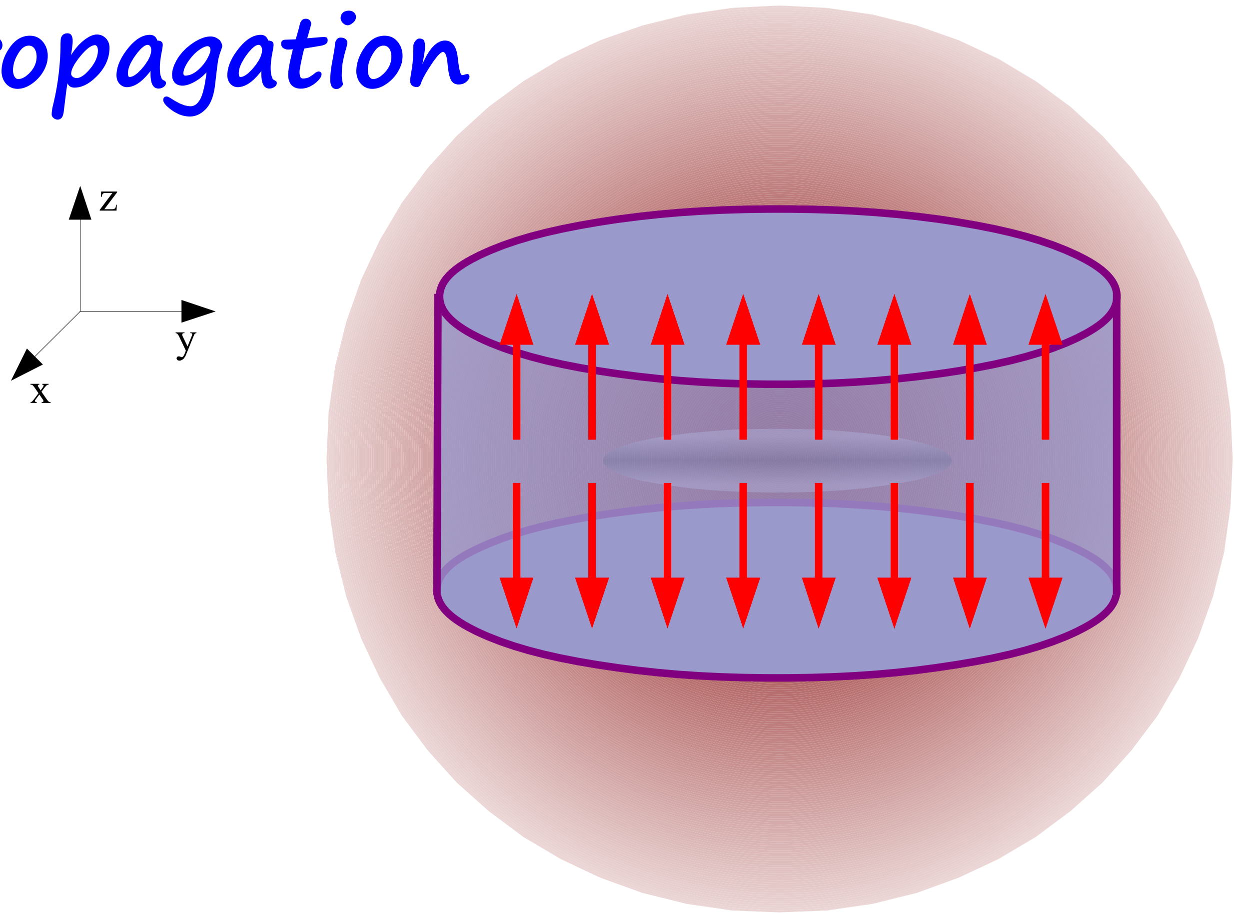
Negligible for positrons.
For antiprotons,

$$\Gamma_{\rm ann} = (n_{\rm H} + 4^{2/3} n_{\rm He})\sigma^{\rm ann}_{\bar{p}p} v_{\bar{p}} \ .$$

$$\sigma^{\rm ann}_{\bar{p}p}(T) = \begin{cases} 661\ (1 + 0.0115\ T^{-0.774} - 0.948\ T^{0.0151})\ \text{mbarn}\ , & T < 15.5\ \text{GeV}\ , \\ 36\ T^{-0.5}\ \text{mbarn}\ , & T \geq 15.5\ \text{GeV}\ , \end{cases}$$

Tan, Ng

Propagation



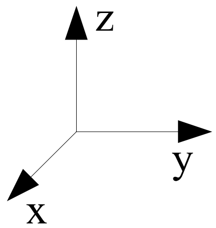
$$0 = \frac{\partial f}{\partial t} = \nabla \cdot [K(T,\vec{r})\nabla f] + \frac{\partial}{\partial T}[b(T,\vec{r}) f] \boxed{- \nabla \cdot [\vec{V}_c(\vec{r}) f]} - 2h\delta(z)\Gamma_{\rm ann} f + Q(T,\vec{r}) \; .$$

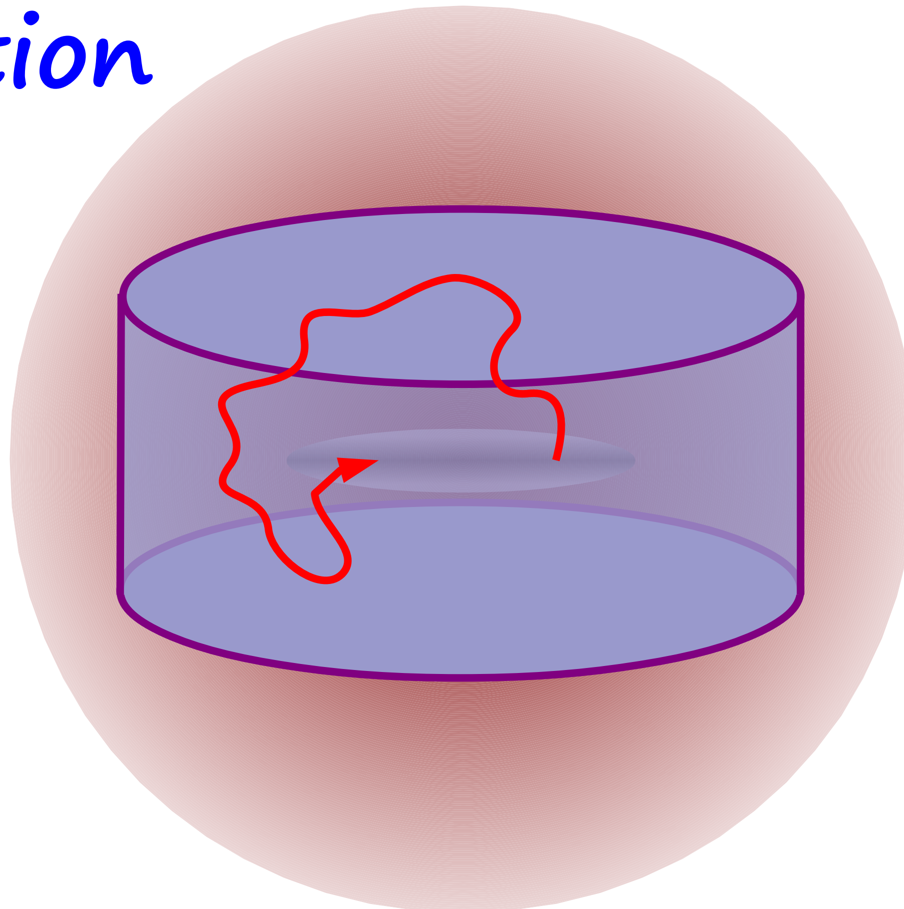
Convection term

- Due to the Milky Way galactic wind.
- It drifts particles away from the Galactic disk.
- Difficult to model. Assume:

$$\vec{V}_c(\vec{r}) \; = \; V_c \; \text{sign}(z) \; \vec{k}$$

Propagation





$$0 = \frac{\partial f}{\partial t} = \nabla \cdot [K(T,\vec{r})\nabla f] - \boxed{\frac{\partial}{\partial T}[b(T,\vec{r})f]} - \nabla \cdot [\vec{V}_c(\vec{r})f] - 2h\delta(z)\Gamma_{\mathrm{ann}} f + Q(T,\vec{r}) \ .$$

Energy loss term

- Due to inverse Compton scattering on the interstellar radiation field (starlight, thermal radiation of dust, CMB) and synchrotron radiation.
- Negligible for antiprotons and antideuterons
- Can be modelled

- Energy loss due to Inverse Compton scattering: $e^+\gamma \to e^+\gamma$

$$b_{\rm ICS}(E_e,\vec{r}) = \int_0^\infty d\epsilon \int_\epsilon^{E_\gamma^{\rm max}} dE_\gamma (E_\gamma - \epsilon)\frac{d\sigma^{\rm IC}(E_e,\epsilon)}{dE_\gamma} f_{\rm ISRF}(\epsilon,\vec{r})$$

Number density of photons in ISRF

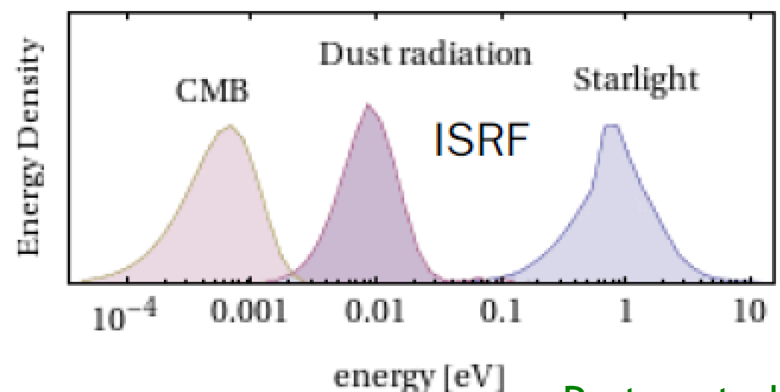
$$\frac{d\sigma^{\rm IC}(E_e,\epsilon)}{dE_\gamma} = \frac{3}{4}\frac{\sigma_{\rm T}}{\gamma_e^2\,\epsilon} \times \left[2q\ln q + 1 + q - 2q^2 + \frac{1}{2}\frac{(q\Gamma)^2}{1+q\Gamma}(1-q)\right]$$

$\gamma_e = E_e/m_e \longrightarrow$ Lorentz factor.

$\Gamma_e = 4\,\gamma_e\,\varepsilon/m_e$

$q = E_\gamma/\Gamma(E_e - E_\gamma)$

$\sigma_T = 0.67$ barn $\longrightarrow$ Compton scattering cross section in the Thomson limit.



Porter et al.

- Energy loss due to synchrotron radiation:

$$b_{\rm sync}(E_e,\vec{r}) = \frac{4}{3}\sigma_T\gamma_e^2\frac{B^2}{2} \qquad B = 6\mu G\exp(-|z|/5{\rm kpc} - r/20{\rm kpc})$$

Approximately $\boxed{b(E) = \frac{E^2}{E_0\tau_E},\ \text{with } E_0 = 1\,\text{GeV and } \tau_E = 10^{16}\text{s}}$

- Energy loss due to Inverse Compton scattering: $e^+\gamma \to e^+\gamma$

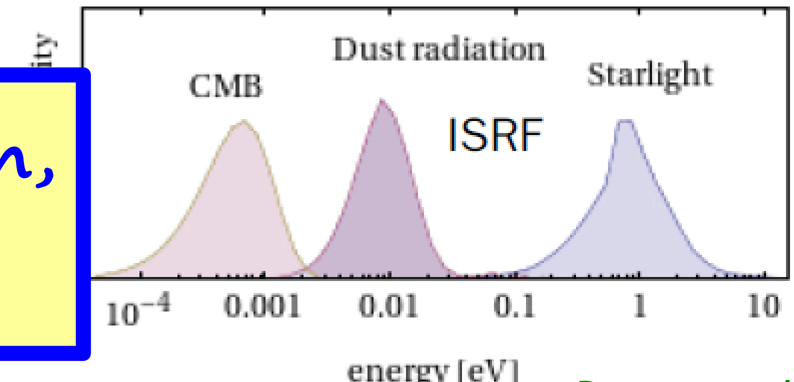
$$b_{\rm ICS}(E_e,\vec{r}) = \int_0^\infty d\epsilon \int_\epsilon^{E_\gamma^{\rm max}} dE_\gamma (E_\gamma - \epsilon) \frac{d\sigma^{\rm IC}(E_e,\epsilon)}{dE_\gamma} f_{\rm ISRF}(\epsilon,\vec{r})$$

Number density
of photons in ISRF

$$\frac{d\sigma^{\rm IC}(E_e,\epsilon)}{dE_\gamma} = \frac{3}{4}\frac{\sigma_{\rm T}}{\gamma_e^2\,\epsilon} \times \left[2q\ln q + 1 + q - 2q^2 + \frac{1}{2}\frac{(q\Gamma)^2}{1+q\Gamma}(1-q)\right]$$

$\gamma_e = E_e/m_e \longrightarrow$ Lorent
$\Gamma_e = 4\,\gamma_e\,\varepsilon/m_e$
$q = E_\gamma/\Gamma(E_e - E_\gamma)$
$\sigma_T = 0.67$ barn $\longrightarrow$ Compton scattering cross section in the Thomson limit.

Not very well known, though...



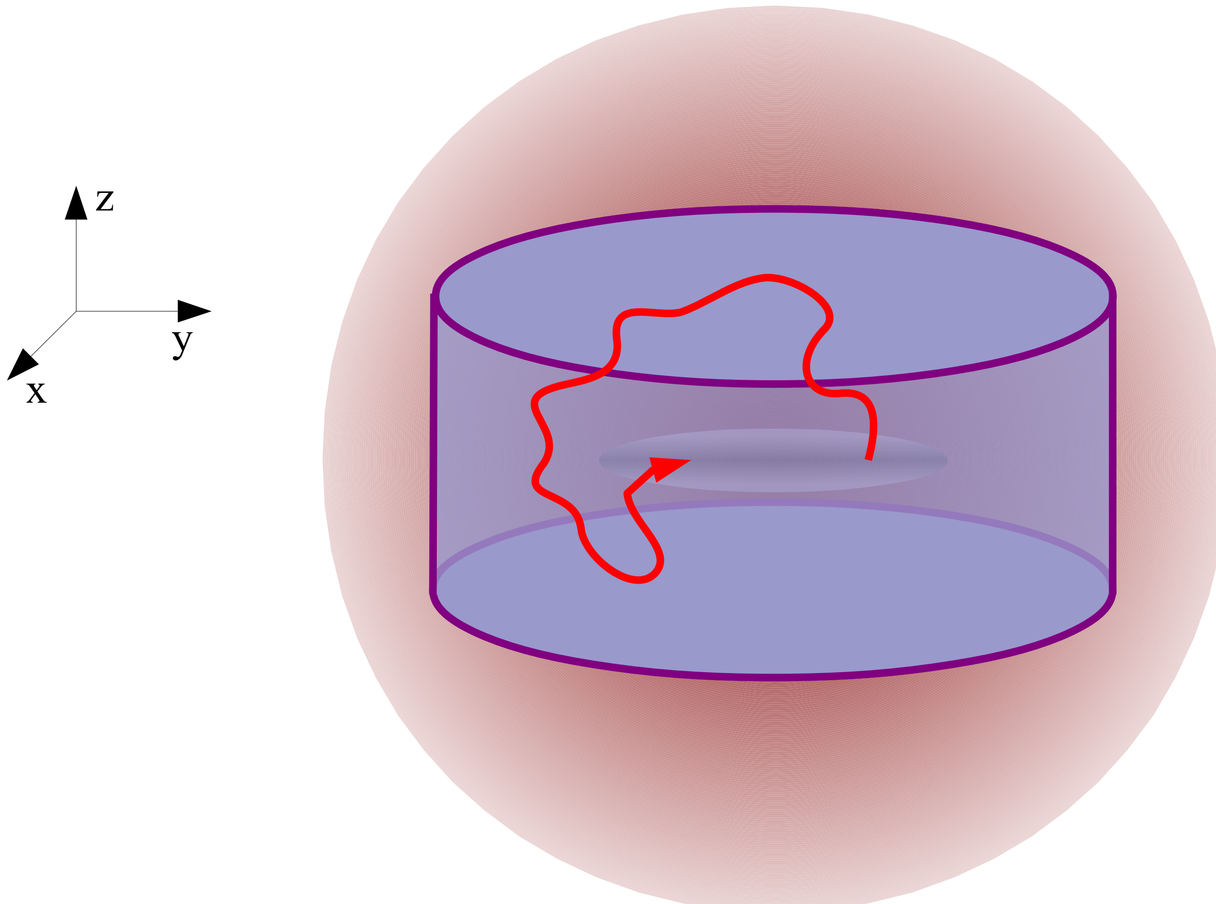
Porter et al.

- Energy loss due to synchrotron radiation:

$$b_{\rm sync}(E_e,\vec{r}) = \frac{4}{3}\sigma_T \gamma_e^2 \frac{B^2}{2}$$

$$B = 6\mu G \exp(-|z|/5{\rm kpc} - r/20{\rm kpc})$$

Approximately $b(E) = \dfrac{E^2}{E_0 \tau_E}$, with $E_0 = 1\,{\rm GeV}$ and $\tau_E = 10^{16}{\rm s}$



$$0 = \frac{\partial f}{\partial t} = \boxed{\nabla \cdot [K(T,\vec{r})\nabla f]} + \frac{\partial}{\partial T}[b(T,\vec{r})f] - \nabla \cdot [\vec{V}_c(\vec{r})f] - 2h\delta(z)\Gamma_{\rm ann} f + Q(T,\vec{r}) \; .$$

Diffusion term

- Due to the tangled magnetic field of the Galaxy.
- Difficult to model. Assume

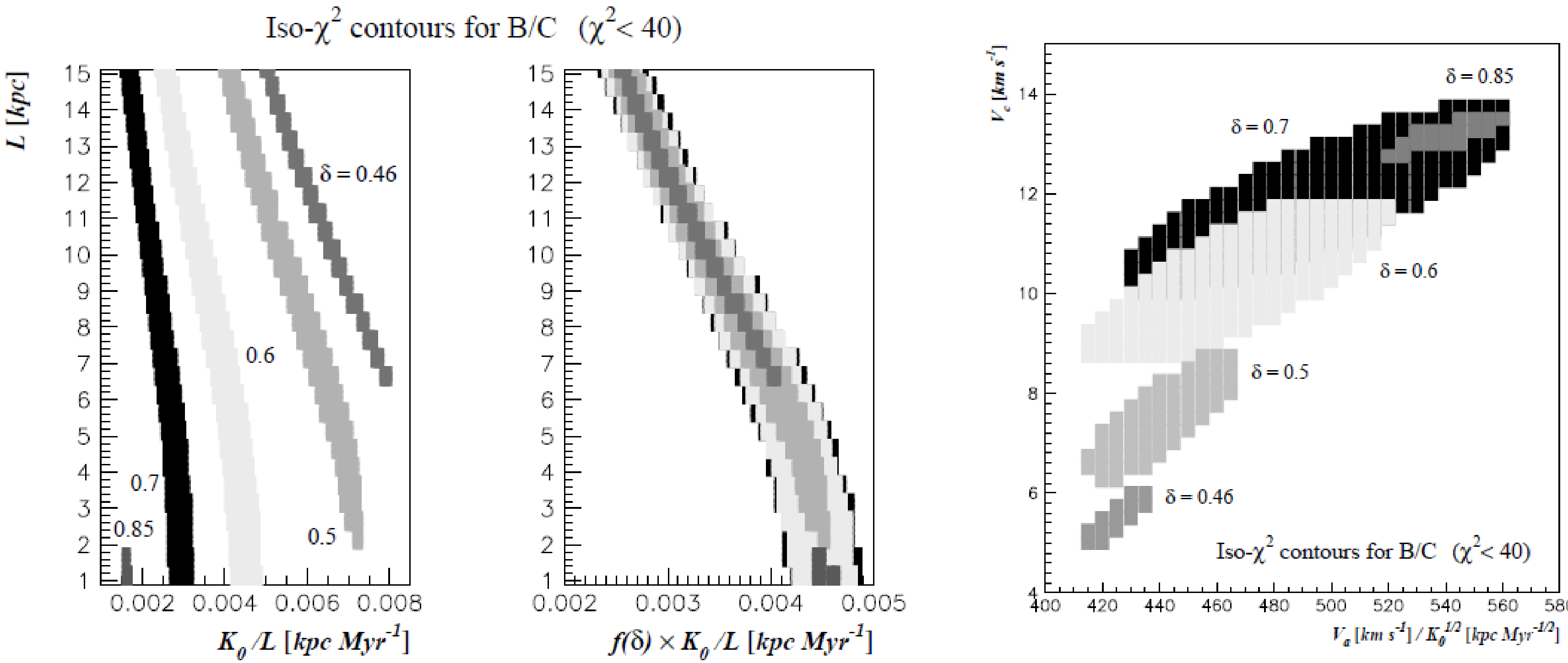
$$K(T) = K_0 \, \beta \, \mathcal{R}^\delta$$

β = velocity
$\mathcal{R}$ = rigidity

$$0 = \frac{\partial f}{\partial t} = \boxed{\nabla \cdot [K(T,\vec{r})\nabla f]} - \frac{\partial}{\partial T}[b(T,\vec{r})f] \boxed{- \nabla \cdot [\vec{V}_c(\vec{r})f]} - 2h\delta(z)\Gamma_{\rm ann} f + Q(T,\vec{r}) \; .$$

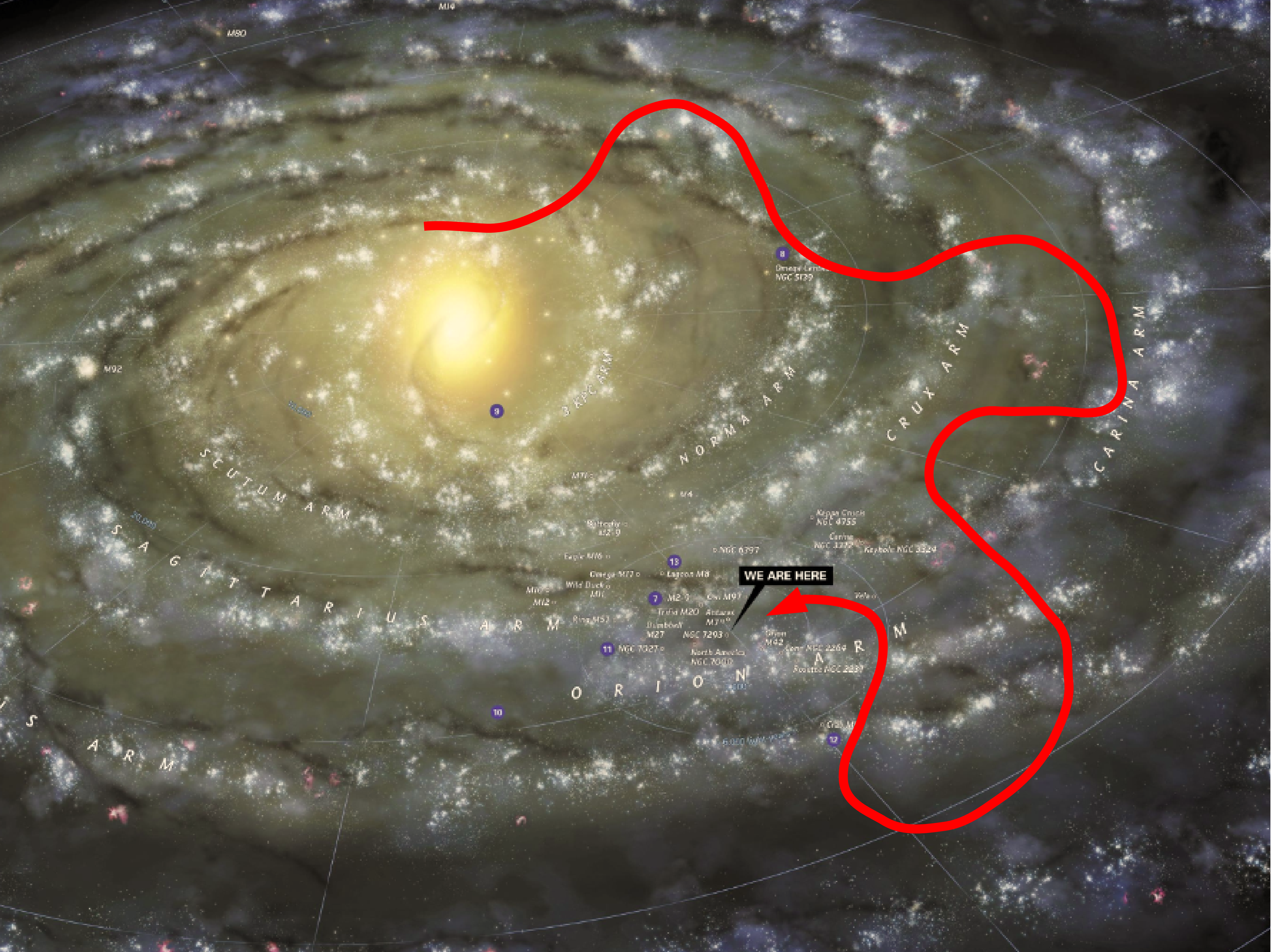
$$K(T) = K_0 \; \beta \; \mathcal{R}^\delta \qquad\qquad \vec{V}_c(\vec{r}) \; = \; V_c \; \text{sign}(z) \; \vec{k}$$

K_0, δ, V_c (as well as L) must be determined with measurements of other cosmic ray species (mainly B/C ratio).



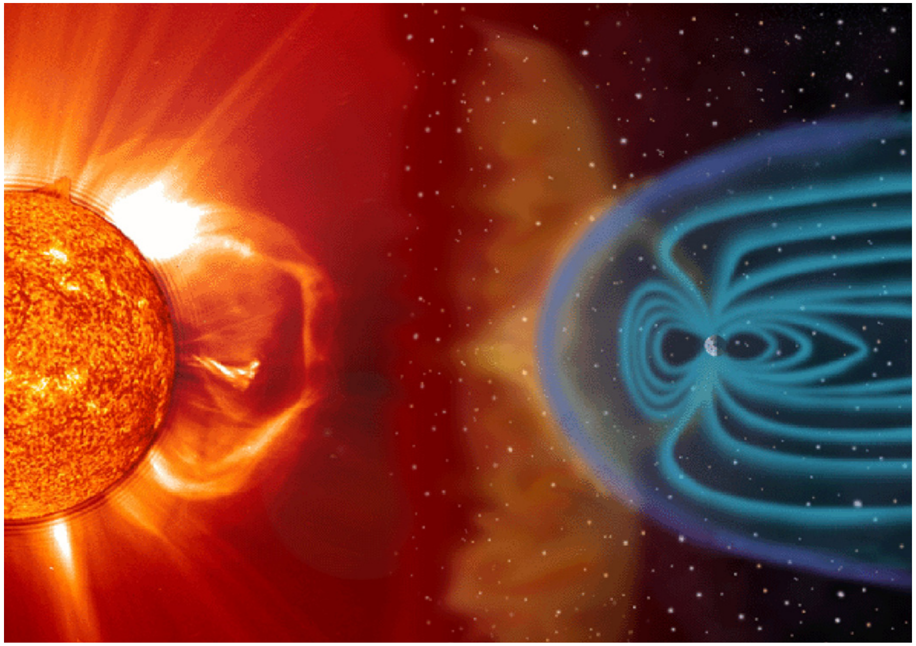
Model	δ	K_0 (kpc^2/Myr)	L (kpc)	V_c (km/s)
MIN	0.85	0.0016	1	13.5
MED	0.70	0.0112	4	12
MAX	0.46	0.0765	15	5

Maurin, Donato, Taillet, Salati '01



WE ARE HERE
SCUTUM ARM
SAGITTARIUS ARM
NORMA ARM
CRUX ARM
CARINA ARM
ORION ARM
3 KPC ARM
Omega Centauri NGC 5139
Kappa Crucis NGC 4755
Keyhole NGC 3324
NGC 6397
Lagoon M8
Eagle M16
Omega M17
Wild Duck M11
Trifid M20
Dumbbell M27
NGC 7293
NGC 7027
North America NGC 7000
Orion M42
Rosette NGC 2237
Ring M57
M80
M92
M4
M12

Propagation inside the Solar System



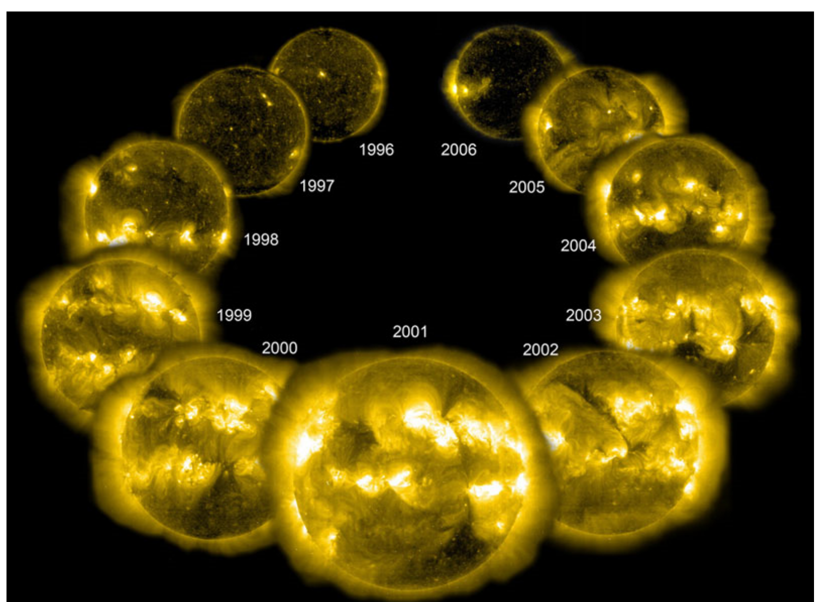
In the "force field approximation",the flux at the top of the atmosphere (TOA) is related to the interstellar flux (IS) by

$$\Phi_{e^\pm}^{\mathrm{TOA}}(E_{\mathrm{TOA}}) = \frac{E_{\mathrm{TOA}}^2}{E_{\mathrm{IS}}^2}\,\Phi_{e^\pm}^{\mathrm{IS}}(E_{\mathrm{IS}})$$

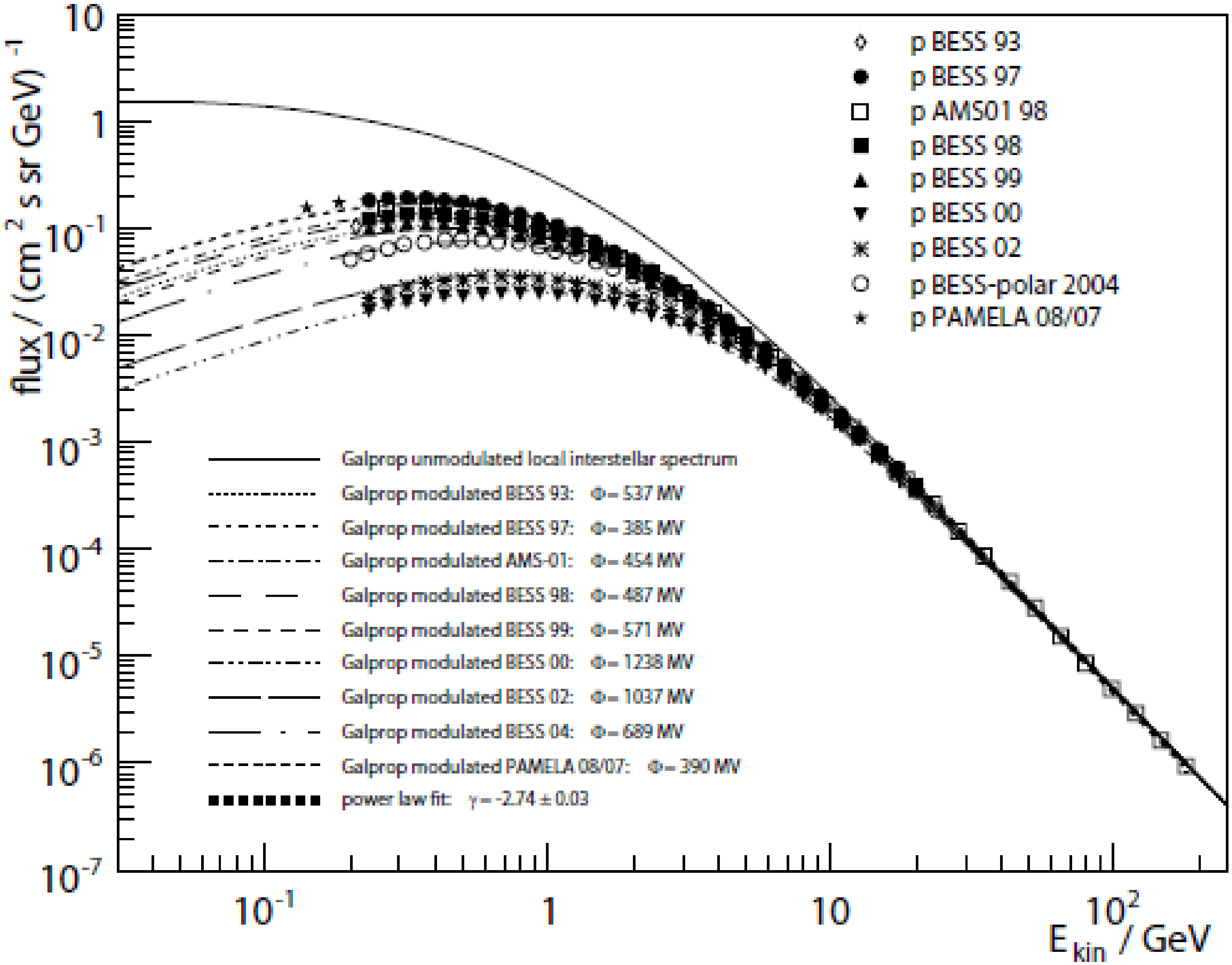
$$E_{\mathrm{IS}} = E_{\mathrm{TOA}} + \phi_F$$

solar modulation parameter

ϕ_F=500 MV – 1.3 GV



Cosmic ray proton spectrum as measured by BESS, AMS-01 and PAMELA



Gast, Schael '09