♠ *Two Topics*

1. Unitarity in W–b scattering

2. $\boxed{\text{Almost degenerate Higgses}}$

Almost degenerate Higgses

It's about production of neutral Higgs bosons

* based on J. R. Ellis, JSL, A. Pilaftsis, PRD70 (2004) 075010 [hep-ph/0404167]

♠ Contents

We wish to demonstrate how to handle a system of almost degenerate spin-0 particles by taking CPV MSSM

- CPV MSSM Higgs Sector
- Coupled-Channel Analysis
- Results
- Summary

♠ *Introduction* : CP Phases in the MSSM

- In Minimal Supersymmetric Standard Model :

  Only double the number of particles $\bigoplus$ Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

$$H_1^0 = \frac{1}{\sqrt{2}}(\phi_1 + i a_1)\,, H_2^0 = \frac{1}{\sqrt{2}}(\phi_2 + i a_2) \;\; ; \; \langle\phi_1\rangle = v\cos\beta\,, \langle\phi_2\rangle = v\sin\beta$$

* Two charged Higgs-boson states H^+ and H^-

* Three neutral states $h[+]$, $H[+]$, and $A[-]$ at the tree level

$$\begin{aligned} A &= a_1 \sin\beta + a_2 \cos\beta \\ \begin{pmatrix} h \\ H \end{pmatrix} &= \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_1 \end{pmatrix} \end{aligned}$$

♠ *Introduction* : CP Phases in the MSSM

- Phenomenology of the MSSM $\Rightarrow$ Probing soft SUSY breaking terms

$$
\begin{aligned}
&(M_3\,\widetilde{g}\widetilde{g} + M_2\,\widetilde{W}\widetilde{W} + M_1\,\widetilde{B}\widetilde{B} + \text{c.c.})\\
&+(\widetilde{u}_R^*\,\mathbf{A_u}\,\widetilde{Q}H_2 - \widetilde{d}_R^*\,\mathbf{A_d}\,\widetilde{Q}H_1 - \widetilde{e}_R^*\,\mathbf{A_e}\,\widetilde{L}H_1 + \text{c.c.})\\
&-\widetilde{Q}^\dagger\,\mathbf{M^2_{\widetilde{Q}}}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{M^2_{\widetilde{L}}}\,\widetilde{L} - \widetilde{u}_R^*\,\mathbf{M^2_{\widetilde{u}}}\,\widetilde{u}_R - \widetilde{d}_R^*\,\mathbf{M^2_{\widetilde{d}}}\,\widetilde{d}_R - \widetilde{e}_R^*\,\mathbf{M^2_{\widetilde{e}}}\,\widetilde{e}_R\\
&- m_2^2 H_2^* H_2 - m_1^2 H_1^* H_1 - (m_{12}^2 H_1 H_2 + \text{c.c.})
\end{aligned}
$$

$$\boxed{\text{Complex Parameters !}}$$

* Supersymmetric complex parameter μ

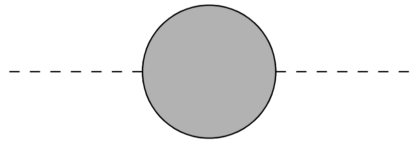
– Physical observables depend on : $\arg(M_i\,\mu\,(m_{12}^2)^*)$ and $\arg(\mathbf{A_f}\,\mu\,(m_{12}^2)^*)$

M. Dugan, B. Grinstein and L. J. Hall, Nucl. Phys. B **255** (1985) 413; S. Dimopoulos and S. Thomas, Nucl. Phys. B **465** (1996) 23

♠ *Introduction* : Radiative Higgs-sector CP Violation

- Effects of non-vanishing CP phases on the Higgs sector

$\tilde{f}_1\,,\tilde{f}_2$

$a_1\,,a_2$  $\phi_1,\ \phi_2$ $\quad \propto \quad \dfrac{3m_f^2}{16\pi^2}\,\dfrac{\Im\mathrm{m}(A_f\,\mu)}{(m_{\tilde{f}_2}^2 - m_{\tilde{f}_1}^2)}$

Mixing among CP–odd and CP–even states !

A. Pilaftsis, PRD**58** 096010 (1998); Phys. Lett. **B435** 88 (1998)

D.A. Demir, PRD**60** 055006 (1999)

A. Pilaftsis and C.E.M. Wagner, NP**B553** 3 (1999)

S. Y. Choi, M. Drees, and JSL, PL**B481** 57 (2000)

M. Carena, J. Ellis, A. Pilaftsis, C.E.M. Wagner, NP**B586** 92 (2000); NP**B625** 345 (2002)

$*$ Renormalization-Group-improved effective potential method $\oplus$ Higgs-boson pole masses

♠ *Introduction* : Radiative Higgs-sector CP Violation

- Consequences of CP-violating mixing among three neutral Higgs bosons

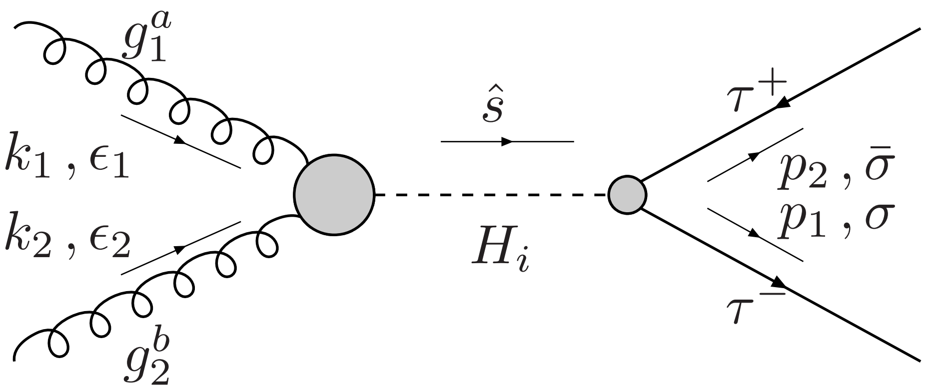
  * The neutral Higgs bosons do not have to carry any definite CP parity

  * 3×3 mixing matrix $O_{\alpha i}$: $(\phi_1, \phi_2, a)^T = O_{\alpha i}(H_1, H_2, H_3)^T$

  * Couplings of the Higgs bosons to SM and SUSY particles are significantly modified

♠ *Collider Signatures* : Processes under consideration

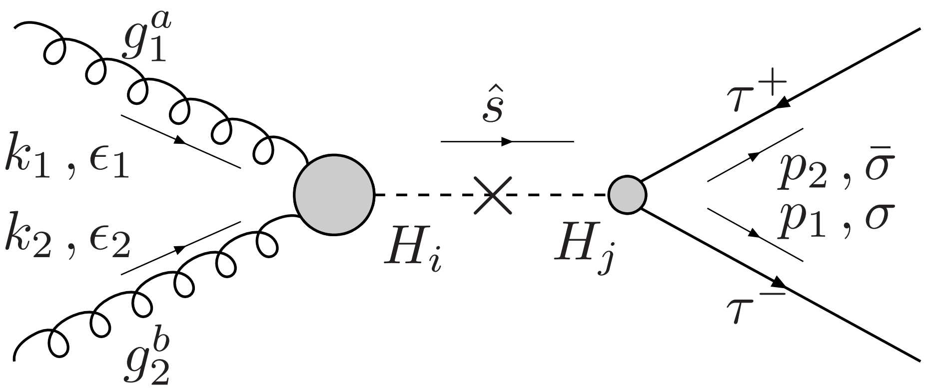
- $gg \to H \to \tau^+_{R,L}\tau^-_{R,L}$



$$\frac{i}{\hat{s} - M_{H_i}^2 + i\Im\mathrm{m}\widehat{\Pi}_{ii}(\hat{s})}$$

♠ *Collider Signatures* : Processes under consideration

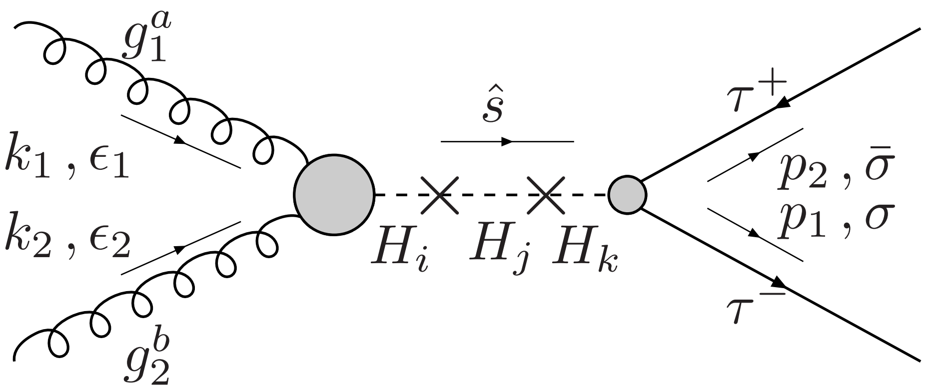
- $gg \to H \to \tau^+_{R,L}\,\tau^-_{R,L}$



$$\frac{i}{\hat{s}-M_{H_i}^2+i\Im\mathrm{m}\widehat{\Pi}_{ii}(\hat{s})}\; i\Im\mathrm{m}\widehat{\Pi}_{ij}(\hat{s})\; \frac{i}{\hat{s}-M_{H_j}^2+i\Im\mathrm{m}\widehat{\Pi}_{jj}(\hat{s})}$$

♠ *Collider Signatures* : Processes under consideration

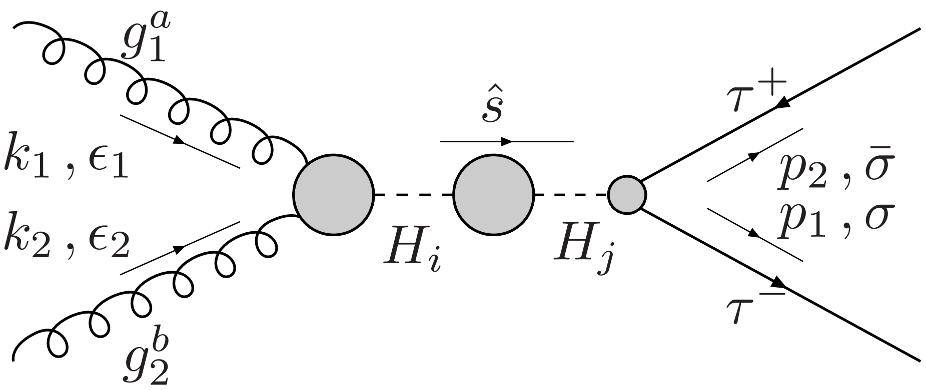
- $gg \to H \to \tau^+_{R,L}\tau^-_{R,L}$



$$\frac{i}{\hat{s}-M^2_{H_i}+i\Im\mathrm{m}\widehat{\Pi}_{ii}(\hat{s})}\; i\Im\mathrm{m}\widehat{\Pi}_{ij}(\hat{s})\;\frac{i}{\hat{s}-M^2_{H_j}+i\Im\mathrm{m}\widehat{\Pi}_{jj}(\hat{s})}\; i\Im\mathrm{m}\widehat{\Pi}_{jk}(\hat{s})\;\frac{i}{\hat{s}-M^2_{H_k}+i\Im\mathrm{m}\widehat{\Pi}_{kk}(\hat{s})}$$

♠ *Collider Signatures* : Processes under consideration

- $gg \to H \to \tau^+_{R,L}\,\tau^-_{R,L}$



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♠ *Collider Signatures* : Coupled-Channel Analysis

- Full 3×3 Higgs-Boson Propagator Matrix: $D(\hat{s})$

In general, the neutral Higgs bosons can not be treated separately !

$$\hat{s}\begin{pmatrix} \hat{s}-M_{H_1}^2+i\Im\mathrm{m}\widehat{\Pi}_{11}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{12}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{13}(\hat{s}) \\ i\Im\mathrm{m}\widehat{\Pi}_{21}(\hat{s}) & \hat{s}-M_{H_2}^2+i\Im\mathrm{m}\widehat{\Pi}_{22}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{23}(\hat{s}) \\ i\Im\mathrm{m}\widehat{\Pi}_{31}(\hat{s}) & i\Im\mathrm{m}\widehat{\Pi}_{32}(\hat{s}) & \hat{s}-M_{H_3}^2+i\Im\mathrm{m}\widehat{\Pi}_{33}(\hat{s}) \end{pmatrix}^{-1}$$

- M_{H_i} : One-loop Higgs-boson pole mass
- $\Im\mathrm{m}\widehat{\Pi}_{ij}(\hat{s})$: absorptive parts of the Higgs-boson self-energies

$$\to \Im\mathrm{m}\widehat{\Pi}_{ii}(\hat{s}=M_{H_i}^2) = M_{H_i}\Gamma_{H_i}$$

♠ *Collider Signatures* : Coupled-Channel Analysis

- Large $\tan\beta$ and $M_{H^\pm}^{\rm pole} \sim 150$ GeV $\Rightarrow$ Tri-mixing scenario J. Ellis, JSL and A. Pilaftsis, PRD70(2004)075010; NPB718(2005)247;PRD71(2005)075007

$$\tan\beta = 50, \quad M_{H^\pm}^{\rm pole} = 155 \ \ {\rm GeV},$$

$$M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = 0.5 \ \ {\rm TeV},$$

$$|\mu| = 0.5 \ \ {\rm TeV}, \quad |A_{t,b,\tau}| = 1 \ \ {\rm TeV}, \quad |M_{1,2}| = 0.3 \ \ {\rm TeV}, |M_3| = 1 \ \ {\rm TeV},$$

$$(\Phi_\mu = 0^\circ), \quad \Phi_{1,2} = 0^\circ, \quad \underline{(\Phi_3\,, \Phi_{A_t,A_b,A_\tau}) = {\rm Varying}}$$

For example, when $(\Phi_3\,,\Phi_A) = (-10^\circ\,, 90^\circ)$, we have (in GeV)

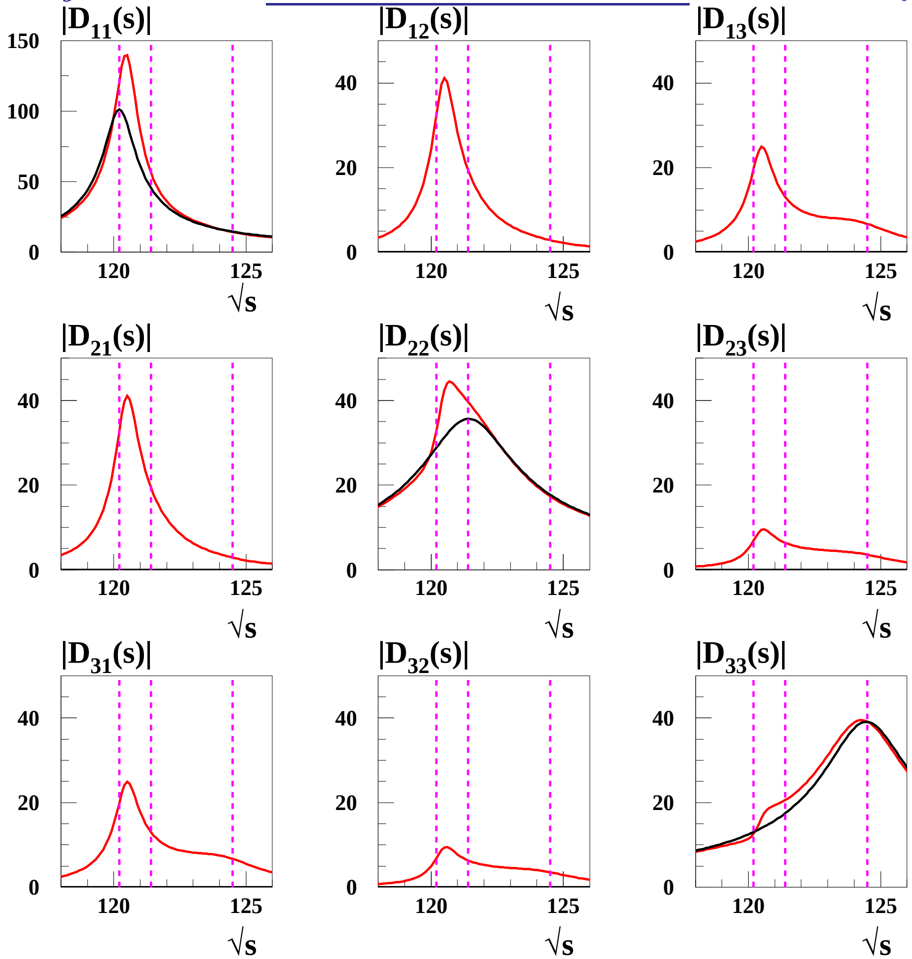
$$M_{H_1} = 120.2, \quad M_{H_2} = 121.4, \quad M_{H_3} = 124.5,$$

$$\Gamma_{H_1} \ = 1.19, \qquad \Gamma_{H_2} \ = 3.42, \qquad \Gamma_{H_3} = 3.20.$$

All Higgs bosons are nearly degenerate with $\Gamma_i \sim \Delta M_H$!

♠ *Collider Signatures* : Coupled-Channel Analysis

- $|D_{ij}(s)|$ for tri-mixing scenario with $\Phi_{A_t,A_b,A_\tau} = 90^\circ$ and $\Phi_3 = -10^\circ$



Set $i\Im\mathrm{m}\widehat{\Pi}_{ij} = 0$ when $i \neq j$

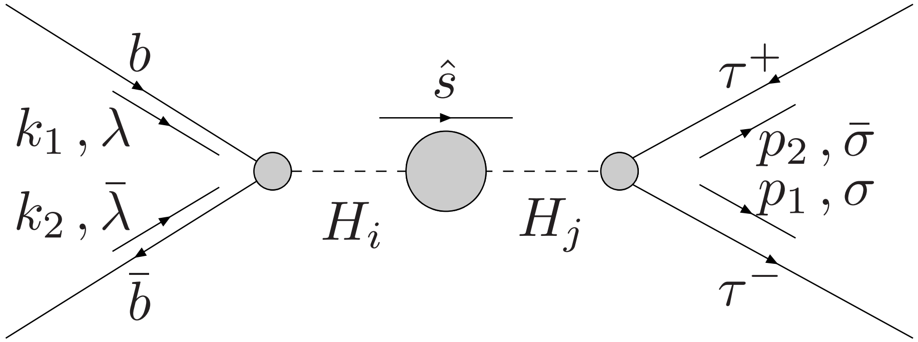
Absorptive parts fully considered

JSL, hep-ph/0409020

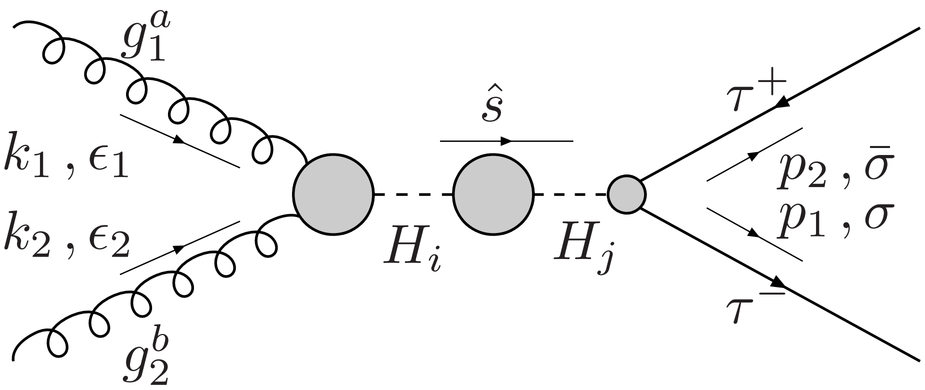
$*$ $D_{ij}(s)$: Propagator Matrix

♠ *Collider Signatures* : The LHC - Inclusive Processes

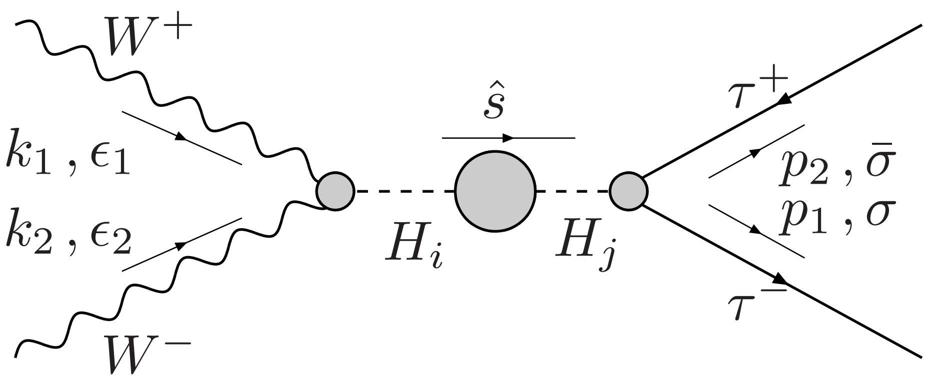
- $b\bar{b} \to H \to \tau^+_{R,L}\tau^-_{R,L}$



- $gg \to H \to \tau^+_{R,L}\tau^-_{R,L}$



- $W^+W^- \to H \to \tau^+_{R,L}\tau^-_{R,L}$



$$\sigma_{\rm tot} = \sigma_{RR} + \sigma_{LL} \, ; \qquad \tau \frac{d\sigma_{\rm tot}}{d\tau}$$

$$\Delta\sigma_{\rm CP} = \sigma_{RR} - \sigma_{LL} \, ; \qquad \tau \frac{d\Delta\sigma_{\rm CP}}{d\tau}$$

$$\sigma_{RR} \equiv \sigma(pp \to H \to \tau^+_R \tau^-_R X)$$

$$\sigma_{LL} \equiv \sigma(pp \to H \to \tau^+_L \tau^-_L X)$$

♠ *Collider Signatures* : The LHC - Inclusive Processes

- Amplitudes

  - b-quark fusion $\beta_f = \sqrt{1 - 4m_f^2/\hat{s}}$

$$\mathcal{M}^{b\bar{b}}(\sigma\bar{\sigma};\lambda\bar{\lambda}) = -\frac{g^2 m_b m_\tau}{4M_W^2}\langle\sigma;\lambda\rangle_b \delta_{\sigma\bar{\sigma}}\delta_{\lambda\bar{\lambda}}$$

$$\langle\sigma;\lambda\rangle_b \equiv \sum_{i,j=1,2,3} (\lambda\beta_b\, g^S_{H_i\bar{b}b} + i g^P_{H_i\bar{b}b})\, D_{ij}(\hat{s})\, (\sigma\beta_\tau\, g^S_{H_j\tau^+\tau^-} - i g^P_{H_j\tau^+\tau^-})$$

  - gluon fusion

$$\mathcal{M}^{gg}(\sigma\bar{\sigma};\lambda_1\lambda_2) = \frac{g\alpha_s m_\tau\sqrt{\hat{s}}\delta^{ab}}{8\pi v M_W}\langle\sigma;\lambda_1\rangle_g \delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2}$$

$$\langle\sigma;\lambda\rangle_g \equiv \sum_{i,j=1,2,3} [S^g_i(\sqrt{\hat{s}}) + i\lambda P^g_i(\sqrt{\hat{s}})]\, D_{ij}(\hat{s})\, (\sigma\beta_\tau g^S_{H_j\tau^+\tau^-} - i g^P_{H_j\tau^+\tau^-})$$

– $W^{\pm}$ fusion $\omega(\pm) = 1$ and $\omega(0) = -k_1 \cdot k_2 / \sqrt{k_1^2 k_2^2}$

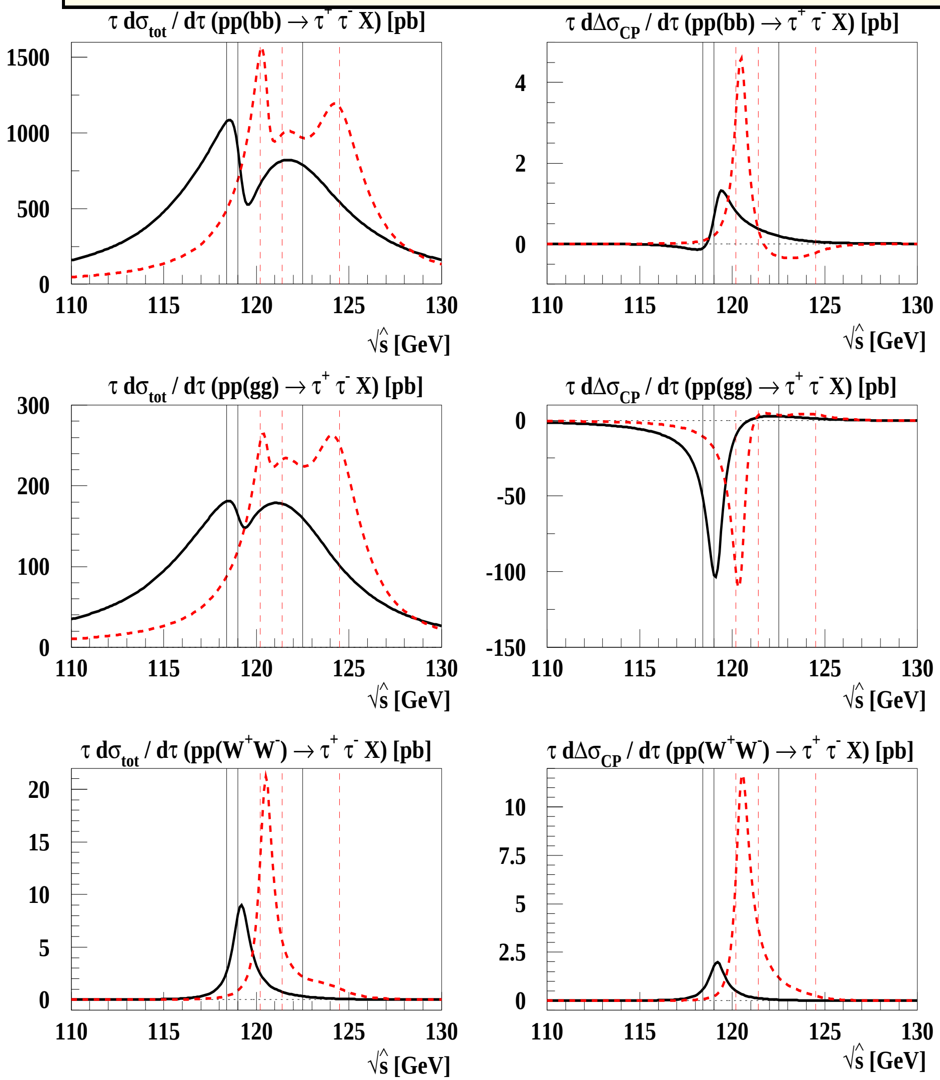
$$\begin{aligned}
\mathcal{M}^{WW}(\sigma\bar{\sigma};\lambda_1\lambda_2) &= \frac{g^2 m_\tau}{2\sqrt{\hat{s}}} \langle \sigma;\lambda_1\rangle_W \delta_{\sigma\bar{\sigma}}\delta_{\lambda_1\lambda_2} \\
\langle \sigma;\lambda\rangle_W &\equiv \sum_{i,j=1,2,3} \omega(\lambda)\, g_{H_i VV}\, D_{ij}(\hat{s})\, (\sigma\beta_\tau\, g^S_{H_j\tau^+\tau^-} - i g^P_{H_j\tau^+\tau^-})
\end{aligned}$$

with $\omega(\pm) = 1$ and $\omega(0) = -k_1 \cdot k_2 / \sqrt{k_1^2 k_2^2}$

♠ *Collider Signatures* : The LHC - Inclusive Processes

$(\Phi_A\,,\Phi_3) = (90^\circ\,,-90^\circ)$

$(\Phi_A\,,\Phi_3) = (90^\circ\,,-10^\circ)$



$\Leftarrow$ b – quark fusion

$\Leftarrow$ gluon fusion

$\Leftarrow$ $W^\pm$ fusion

♠ *Summary*

- We present the general formalism for studying the production, mixing and decay of a coupled system of neutral Higgs bosons at high-energy colliders

- You should not expect the simple Breit Wigner line shape if there are more than two resonances intefering each other

- The event line shape strongly depends on the production mechanism and the decay process

$\boxed{\text{BACKUP}}$

The absorptive part of the Higgs-boson propagator matrix receives contributions from loops of fermions, vector bosons, associated pairs of Higgs and vector bosons, Higgs-boson pairs, and sfermions:

$$\Im\mathrm{m}\widehat{\Pi}_{ij}(s) = \Im\mathrm{m}\widehat{\Pi}_{ij}^{ff}(s) + \Im\mathrm{m}\widehat{\Pi}_{ij}^{VV}(s) + \Im\mathrm{m}\widehat{\Pi}_{ij}^{HV}(s) + \Im\mathrm{m}\widehat{\Pi}_{ij}^{HH}(s) + \Im\mathrm{m}\widehat{\Pi}_{ij}^{\tilde{f}\tilde{f}}(s)\,. \tag{1}$$

The contributions of the exchanges of the bottom and top quarks, τ leptons, neutralinos χ_i^0 and charginos χ_i^+ are summed in $\Im\mathrm{m}\widehat{\Pi}_{ij}^{ff}(s)$. The latter may conveniently be cast into the form

$$\begin{aligned}\Im\mathrm{m}\widehat{\Pi}_{ij}^{ff}(s) \quad &= \quad \frac{s}{8\pi} \sum_{f,f'=b,t,\tau,\tilde{\chi}^0,\tilde{\chi}^-} K_f(s)\, g_f^2 \Delta_{ff'} N_C^f \Big[(1-\kappa_f-\kappa_{f'})(g^S_{H_i\bar{f}'f} g^{S*}_{H_j\bar{f}'f} + g^P_{H_i\bar{f}'f} g^{P*}_{H_j\bar{f}'f}) \\ &\qquad -2\sqrt{\kappa_f\kappa_{f'}}(g^S_{H_i\bar{f}'f} g^{S*}_{H_j\bar{f}'f} - g^P_{H_i\bar{f}'f} g^{P*}_{H_j\bar{f}'f}) \Big]\, \lambda^{1/2}(1,\kappa_f,\kappa_{f'})\, \Theta\left(s-(m_f+m_{f'})^2\right), \end{aligned} \tag{2}$$

where $K_{b,t}(s) \simeq 1 + 5.67\frac{\alpha_S(s)}{\pi}$, $\Delta_{ff'} = \delta_{ff'}\,(f,f' = b,t,\tau)$, $\frac{4}{1+\delta_{ff'}}\,(f,f' = \tilde{\chi}^0_{1,2,3,4})$, or $1\,(f,f' = \tilde{\chi}^-_{1,2})$, and $\lambda(x,y,z) = x^2+y^2+z^2-2(xy+yz+zx)$ with $\kappa_x \equiv m_x^2/s$. Here and subsequently, we follow the convention of `CPsuperH` for the couplings of the Higgs bosons to fermions, vector bosons, Higgs bosons, and sfermions.

The vector-boson loop contributions are

$$
\begin{aligned}
\Im\mathrm{m}\widehat{\Pi}^{VV}_{ij}(s) &= \frac{g^2 g_{H_iVV}\, g_{H_jVV}\,\delta_V}{128\pi M_W^2}\beta_V \Big[-4M_V^2(2s-3M_V^2) \\
&\qquad +2M_V^2(M_{H_i}^2+M_{H_j}^2)+M_{H_i}^2 M_{H_j}^2\Big]\,\Theta(s-4M_V^2), \qquad (3)
\end{aligned}
$$

where $\beta_V = (1-4\kappa_V)^{1/2}$ and $\delta_W = 2$, $\delta_Z = 1$.

Correspondingly, the exchanges of Higgs and vector boson pairs give

$$
\begin{aligned}
\Im\mathrm{m}\widehat{\Pi}^{HV}_{ij}(s) &= \frac{g^2}{64\pi M_W^2}\sum_{k=1,2,3} g_{H_iH_kZ}\, g_{H_jH_kZ}\, \lambda^{1/2}(1,\kappa_Z,\kappa_{H_k})\Big[-4sM_Z^2+(M_Z^2-M_{H_k}^2)^2 \\
&\quad +(M_Z^2-M_{H_k}^2)(M_{H_i}^2+M_{H_j}^2)+M_{H_i}^2M_{H_j}^2\Big]\,\Theta\left(s-(M_Z+M_{H_k})^2\right) \\
&+ \frac{g^2}{32\pi M_W^2}\Re\mathrm{e}(g_{H_iH^+W^-}\, g^*_{H_jH^+W^-})\lambda^{1/2}(1,\kappa_W,\kappa_{H^\pm})\Big[-4sM_W^2+(M_W^2-M_{H^\pm}^2)^2 \\
&\quad +(M_W^2-M_{H^\pm}^2)(M_{H_i}^2+M_{H_j}^2)+M_{H_i}^2M_{H_j}^2\Big]\,\Theta\left(s-(M_W+M_{H^\pm})^2\right). \qquad (4)
\end{aligned}
$$

In deriving (3) and (4), we apply the PT to the MSSM Higgs sector following a procedure very analogous to the one given in J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **80** (1998) 2785; Phys. Rev. D **58** (1998) 053002 for the SM Higgs sector. As a consequence, the

PT self-energies $\Im\mathrm{m}\widehat{\Pi}^{VV}_{ij}(s)$ and $\Im\mathrm{m}\widehat{\Pi}^{VH}_{ij}(s)$ depend linearly on s at high energies. This differs crucially from the bad high-energy dependence $\propto s^2$ that one usually encounters when the Higgs-boson self-energies are calculated in the unitary gauge. In fact, if the Higgs-boson self-energies are embedded in a truly gauge-independent quantity such as the S-matrix element of a $2 \to 2$ process, the badly high-energy-behaved s^2-dependent terms cancel against corresponding s^2 terms present in the vertices and boxes order by order in perturbation theory. In this context, PT provides a self-consistent approach to extract those s^2-dependent terms from boxes and vertices, thus giving rise to effective Higgs self-energies that are independent of the gauge-fixing parameter and s^2.

Finally, the contributions of the MSSM Higgs bosons and sfermions are

$$\Im\mathrm{m}\widehat{\Pi}^{HH}_{ij}(s) = \frac{v^2}{16\pi} \sum_{k\geq l=1,2,3} \frac{S_{ij;kl}}{1+\delta_{kl}} g_{H_iH_kH_l} g_{H_jH_kH_l} \lambda^{1/2}(1,\kappa_{H_k},\kappa_{H_l})\,\Theta\left(s-(M_{H_k}+M_{H_l})^2\right)\,, \tag{5}$$

$$\Im\mathrm{m}\widehat{\Pi}^{\tilde{f}\tilde{f}}_{ij}(s) = \frac{v^2}{16\pi} \sum_{f=b,t,\tau} \sum_{k,l=1,2} N_C^f\, g_{H_i\tilde{f}^*_k\tilde{f}_l} g^*_{H_j\tilde{f}^*_k\tilde{f}_l} \lambda^{1/2}(1,\kappa_{\tilde{f}_k},\kappa_{\tilde{f}_l})\,\Theta\left(s-(M_{\tilde{f}_k}+M_{\tilde{f}_l})^2\right)\,. \tag{6}$$

Note that the symmetry factor $S_{ij;kl}$ has to be calculated appropriately. When $i=j=1$ and $k=l=2$, for example, the symmetry factor for the squared self-coupling $g^2_{H_1H_2H_2}$ is $S_{11;22}=4$.